No. DP26-10

RCESR Discussion Paper Series

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March 2026

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The Last Broker*

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Abstract

We develop a dynamic theory of housing market collapse in which population decline interacts with information friction to produce irreversible market death. Declining transactions raise valuation uncertainty, eroding broker profitability and eliminating the intermediation channel through which most transactions are completed. We embed Jovanovic (1982)–Hopenhayn (1992) industry dynamics with forward-looking broker value functions and establish three main theorems, each proved in full: a tipping-point theorem characterising the separatrix between functioning and dead-market attractors; a dual-exit acceleration theorem showing that economic and demographic exit interact multiplicatively to compress the collapse timeline; and a welfare theorem establishing that disclosure is socially underprovided, with a convex marginal social benefit. The model delivers sharp monotone comparative statics throughout.

JEL: D83, L11, R21, D21, D92, R23.

Keywords: Housing market death, broker exit, information externalities, tipping points, dual-exit dynamics, akiya crisis.

*The author gratefully acknowledges financial support from the JSPS KAKEN Grant (S) 24H00012. I thank Cristian Badarinza, Tarun Ramadorai, for valuable comments. All errors are my own.

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1 Introduction

Japan's housing market faces an unprecedented structural crisis. Approximately 9 million dwellings stand permanently vacant (MIC 2023), and licensed real estate brokers in non-metropolitan prefectures declined by 18% between 2010 and 2023 (Shimizu 2024). The standard explanation emphasises demographic decline: aging, low fertility, and rural outmigration. Yet Germany, the UK, and many US cities have experienced comparable depopulation without comparable vacancy crises, because their housing markets—supported by robust price information infrastructure—continue to function as allocation mechanisms.

The difference lies in information. In the UK, all residential transaction prices have been publicly recorded since 2000. In Japan, the Ministry of Land, Infrastructure, Transport and Tourism surveys roughly 30% of transactions on a voluntary basis. When transactions thin out and price comparables become scarce, valuation uncertainty rises, buyers and sellers fail to agree on price, and the market enters a self-reinforcing downward spiral. Crucially, this spiral can terminate in an *absorbing state*— market death—from which recovery requires external institutional intervention.

The mechanism. We model collapse as a *double spiral* through two coupled feedback loops. Loop 1 runs through *information*: declining transactions raise the optimal search radius, widening valuation uncertainty, which lowers the matching probability, which discourages participation, which thins transactions further. Loop 2 runs through *intermediation*: falling deal volume and prices compress broker profits, triggering exit; once broker density falls below a critical threshold, the intermediation channel shrinks to a small direct-trading residual, and transactions effectively cease. The loops couple continuously: a sufficiently severe decline in broker density amplifies the information-friction collapse in Loop 1 as well.

Three main results. We establish three theorems, each proved in full.

Theorem 1 (Tipping Point). For any fixed population density ρ and disclosure level θ, the transaction dynamics $T_{t+1} = h(T_t; \theta, \rho)$ admit at most two stable fixed points: a functioning market $T^*(\theta, \rho) > 0$ and a dead market $T = 0$. The basin of attraction of $T = 0$ is $[0, T^{\text{crit}})$, where T^{crit} is strictly decreasing in θ and strictly increasing in ρ. As ρ falls through a critical threshold $\rho^{\text{crit}}(\theta)$, the functioning-market equilibrium ceases to exist and the market collapses irreversibly.

Theorem 2 (Dual-Exit Acceleration). Broker exit operates through two independent channels: a *state-dependent* channel driven by thin markets and low disclosure, and a *time-dependent* channel driven by principal aging. The two channels interact multiplicatively: markets facing both economic deterioration and aging broker populations collapse strictly faster than either channel alone predicts. The interaction effect is amplified by low disclosure.

Theorem 3 (Welfare and Disclosure Underprovision). Define social welfare relative to the dead-market benchmark. Welfare is strictly increasing and convex in disclosure θ: the marginal social gain of disclosure is *increasing* in θ. The market equilibrium undersupplies disclosure because brokers do not internalise the information production externality—each transaction generates comparables that benefit all future participants—nor the market-survival externality.

Additional results. We also characterise the Monopoly Exit Paradox (Proposition 1): as broker density falls to one, the monopoly broker charges the highest feasible commission, yet is still forced to exit when population falls below a critical density that is decreasing in θ. Spatial contagion (Proposition 2) and Global Games equilibrium uniqueness (Proposition 3) are developed in the appendix.

Methodological choices. We deliberately keep the model tractable. Population density ρ is a fixed parameter; population decline is analysed as comparative dynamics across parameter values. We show that the core results hold for a 2-dimensional reduction (T_t, A_t^*) of the full state space and do not require the full 5-dimensional system $(T_t, A_t^*, \bar{p}_t, a_t, Q_t)$ to generate the main theorems. A numerical illustration using a phase diagram is provided in Appendix C to give intuition for the quantitative magnitudes; the theorems themselves are purely analytical.

Organization. Section 2 surveys the related literature. Section 3 develops the model, starting with a complete Primitives and Timing block. Section 4 presents the three theorems and additional results. Section 5 concludes. Appendix A collects proofs of the three propositions (the main theorem proofs appear within the text). Appendix B provides a complete notation table. Appendix C presents the numerical phase-diagram illustration.

2 Related Literature

Our paper combines three strands of literature: housing search and matching, endogenous firm dynamics, and information economics. Table 1 positions our three main theorems relative to the key reference models.

Table 1: Positioning Relative to Key Reference Papers

	Tipping Point	Dual-Exit Accel.	Welfare/ Underprov.	Mono. Paradox
Hopenhayn (1992)				
Wheaton (1990)				
Glaeser and Gyourko (2005)				
Eisfeldt (2004)			✓	
Diamond (1982)	✓			
Morris and Shin (2003)	✓			
Badarinza et al. (2022)				
Andersen et al. (2022)				
This paper	✓	✓	✓	✓

Note: ✓ indicates a paper provides a formal result in the column's dimension. Diamond (1982) and Morris and Shin (2003) establish multiplicity/tipping in search and global-games settings, respectively, but not the housing-market irreversibility result proved here. Eisfeldt (2004) proves disclosure underprovision for a single asset market without the tipping-point or dual-exit channels.

Housing search and matching. Wheaton (1990) establishes that vacancy and prices are jointly determined in a search equilibrium. Ngai and Sheedy (2020) endogenise the moving decision, generating thick-market externalities. Glaeser and Gyourko (2005) document that housing durability creates persistent vacancy in declining cities. Molloy (2016) documents rising long-term housing vacancy in the United States driven by demographic shifts and spatial sorting. Badarinza et al. (2022) structurally estimate a model of capital allocation in decentralised cross-border commercial real estate markets, showing that affinity-based counterparty matching perpetuates gravity relationships in international investment flows; their framework for search frictions in decentralised asset markets directly motivates our broker-matching structure. Andersen et al. (2022) quantify reference dependence and loss aversion in the Danish housing market using a structural model with a sale decision: sellers attach utility to nominal gains over the purchase price, and losses reduce utility roughly 2.5 times more than gains increase it. Their finding that reference-dependent seller behaviour shapes both listing prices and turnover motivates our modelling of the seller's reservation price $\underline{p}_t^O$ as incorporating a reference-point component in addition to the financial cost of waiting.

Our contribution: We characterise the absorbing dead-market state and prove conditions under which the spiral is irreversible—results absent from the existing search literature, which focuses on stationary equilibria.

Firm dynamics. Jovanovic (1982) introduces the selection model of firm exit: firms enter uncertain of their own productivity, learn through operation, and exit when realised efficiency falls below a viability threshold. Hopenhayn (1992) develops the general-equilibrium free-entry framework in which the stationary firm-size distribution and the entry and exit rates are jointly determined by a free-entry condition. Melitz (2003) shows that trade liberalisation reallocates market share toward more productive firms, raising aggregate productivity while reducing average productivity among incumbents—our disclosure policy generates an analogous selection paradox. Kerr et al. (2014) argue that entrepreneurship is fundamentally about experimentation: success probabilities are low, extremely skewed, and unknowable before investment, with deep consequences for the organisational form of innovation. Parker and Belghitar (2006) track a nationally representative panel of American nascent entrepreneurs and show that personal and economic characteristics— including the value of waiting and financial constraints—determine whether nascent entrepreneurs proceed to venture launch, remain in the nascent stage, or give up entirely; their survival-analysis framework motivates our modelling of the broker's exit decision as a dynamic stopping problem.

Our contribution: We introduce *dual-exit dynamics*—the multiplicative interaction between economic and demographic exit—as a new mechanism absent from standard industry-dynamics models, which assume stationary or growing markets.

Information, liquidity, and externalities. Diamond (1982) establishes the foundational search-complementarity result: each agent's decision to enter the market raises the matching probability for all others, generating a coordination externality that supports multiple equilibria—the mechanism underlying our tipping-point structure. Eisfeldt (2004) shows that information production is complementary to trading, generating a positive externality that leads to socially insufficient liquidity. Brunnermeier and Pedersen (2009) formalise the liquidity spiral in financial markets. Hendel et al. (2009) show that mandatory MLS information sharing raises transaction prices and reduces time-on-market.

Our contribution: We combine the information externality of Eisfeldt (2004) with the matching framework of Wheaton (1990) and the firm-dynamics structure of Hopenhayn (1992) to generate the irreversibility result: once the information externality collapses, re-entry is unprofitable and the market cannot self-revive.

3 The Model

3.1 Primitives and Timing

Locations and time. The economy consists of L locations indexed by $\ell \in \mathcal{L}$, each characterised by a fixed population density $\rho(\ell) \in (0, \infty)$ and a housing stock $H(\ell) > 0$. Time is discrete, $t = 0, 1, 2, \ldots$. Population density is a fixed parameter; Theorem 1(iv) studies how equilibrium changes as ρ varies (comparative dynamics).

Agents. Three types of agents operate in each location.

(i) *Old seller O*: owns one housing unit, reservation price $\underline{p}^O \sim F_O(\cdot)$ with mean μ_O and variance σ_O^2.

(ii) *Young buyer Y*: values housing at $V \sim F_Y(\cdot)$ with mean μ_V and variance σ_V^2. Has wealth $W > 0$.

(iii) *Broker b*: characterised by productive type $z_b \sim F_z$ (drawn once at entry) and principal age $\omega_b \in [\underline{\omega}, \bar{\omega}]$ (increases deterministically by 1 per period).

Disclosure. The disclosure parameter $\theta \in [0, 1]$ is the fraction of completed transactions for which the price is publicly recorded. Disclosed transactions constitute the *comparable sales pool* used for property valuation.

Commission regulation. Japan's Real Estate Brokerage Act specifies a *maximum* commission $\bar{\tau}(\bar{p})$ as an increasing function of transaction price $\bar{p}$ (approximately 3% for the price range considered here). Brokers charge $\tau_t(\ell) \leq \bar{\tau}$ chosen to maximise profit subject to this cap.

Within-period timing. In each period t at location ℓ:

1. **Broker decisions.** Each incumbent broker decides whether to remain active. A mass λ^E of potential entrants draw $z_b \sim F_z$ and decide whether to pay entry cost $\kappa^E > 0$.
2. **Information production.** Each agent draws on the pool of $\theta \cdot T_{t-1}(\ell) \cdot \rho(\ell)$ comparable sales per unit area from the previous period to estimate property values.
3. **Participation.** Sellers decide whether to list; buyers decide whether to search.
4. **Commission.** Active brokers set commission $\tau_t \leq \bar{\tau}$ (regime-dependent).
5. **Matching.** Broker b contacts a seller–buyer pair; a transaction occurs if the bid–ask gap falls within a negotiation band.
6. **Prices settled; comparables recorded.** A fraction θ of completed transactions enter the next period's pool.
7. **State update.** T_t, A_t^*, a_t, Q_t updated.

3.2 Notation Summary

A full notation table appears in Appendix B. Key variables:

$T_t(\ell)$	transactions per 1K units per year	$A_t^*(\ell)$	broker density (per 10K pop)
$R_t^*(\ell;\theta)$	agent-optimal search radius	$N_t(R)$	comparable sample size within R
$\sigma_g^2(R;\theta)$	estimation error	$\tilde{\alpha}_t$	per-period matching probability
ϕ_t^S	state-dependent exit rate	ϕ_t^T	time-dependent exit rate
$\bar{\tau}$	legal commission cap	τ_t^*	equilibrium commission
δ^{dir}	direct-channel efficiency	$\bar{p}_t(\ell)$	mean transaction price

3.3 Information Structure: Agent-Chosen Search Radius

Valuation problem. Property j at ℓ has true value $V_{j\ell} = \mathbf{x}'_{j\ell}\boldsymbol{\beta} + h_{j\ell}$, where $\mathbf{x}_{j\ell}$ are observable hedonic attributes and $h_{j\ell}$ is a location-specific quality component that varies smoothly in space at rate σ_h (hedonic drift). An appraiser estimates $V_{j\ell}$ by running a hedonic regression on comparables within a search radius R of j.

Comparable sample size. Given radius R, the number of comparable sales available is:

$$N_t(R,\ell;\theta) = \theta \cdot T_t(\ell) \cdot \rho(\ell) \cdot \pi R^2 \tag{1}$$

This is the product of the disclosed transaction density (per unit area) and the search area πR^2.

Estimation error. The mean squared error of the hedonic estimate decomposes as:

$$\sigma_g^2(R;\theta,T_t,\ell) = \underbrace{n_U(\theta)\bar{\beta}^2\sigma_x^2}_{\text{(A) attribute bias}} + \underbrace{\frac{\sigma_\beta^2}{N_t(R,\ell;\theta)}}_{\text{(B) coefficient uncertainty}} + \underbrace{\sigma_h^2 R^2}_{\text{(C) spatial drift}} \tag{2}$$

Term (A) captures attributes unobservable when $\theta < 1$; term (B) captures estimation uncertainty decreasing in sample size N; term (C) captures increasing bias as comparables are drawn from further away.

Agent-optimal radius. Each agent chooses R to minimise σ_g^2:

$$R_t^*(\ell;\theta) = \underset{R>0}{\operatorname{argmin}}\ \sigma_g^2(R;\theta,T_t,\ell) \tag{3}$$

Taking the first-order condition using $N_t(R) = \theta T_t \rho \pi R^2$:

$$\frac{d\sigma_g^2}{dR} = -\frac{\sigma_\beta^2}{N_t(R)^2} \cdot \frac{dN_t}{dR} + 2\sigma_h^2 R = -\frac{2\sigma_\beta^2}{\theta T_t \rho \pi \, R^3} + 2\sigma_h^2 R = 0 \quad \Rightarrow \quad R_t^* = \left(\frac{\sigma_\beta^2}{\sigma_h^2 \cdot \theta T_t \rho \pi} \right)^{1/4} \tag{4}$$

Lemma 1 (Properties of R_t^* and σ_g^2)**.** *The optimal radius $R_t^*(\ell;\theta)$ and the associated minimised error $\sigma_g^{2*} \equiv \sigma_g^2(R_t^*;\theta,T_t,\ell)$ satisfy:*

(i) R_t^* is strictly decreasing in T_t, ρ, and θ.

(ii) σ_g^{2*} is strictly decreasing in T_t, ρ, and θ.

(iii) $\sigma_g^{2*} \to \infty$ as $T_t \to 0$ (information collapses).

(iv) $\sigma_g^{2*} \to n_U(\theta)\bar{\beta}^2\sigma_x^2 > 0$ as $T_t \to \infty$ (irreducible attribute-bias floor).

(v) $\partial R_t^*/\partial\theta < 0$ and $\partial^2\sigma_g^{2*}/\partial\theta^2 > 0$ (disclosure reduces error at an increasing marginal rate as θ is already high).

Proof. From (4), $R_t^* \propto (\theta T_t \rho)^{-1/4}$, which is strictly decreasing in T_t, ρ, θ. This proves (i). Substituting R_t^* back into (2) gives:

$$\sigma_g^{2*} = n_U(\theta)\bar{\beta}^2\sigma_x^2 + C(\sigma_\beta,\sigma_h)\,(\theta T_t \rho)^{-1/2},$$

where $C = 2(\sigma_\beta^2\sigma_h^2/\pi)^{1/2} > 0$. This expression is strictly decreasing in T_t, ρ, θ, establishing (ii). As $T_t \to 0$, $(\theta T_t\rho)^{-1/2} \to \infty$, giving (iii). As $T_t \to \infty$, $(\theta T_t\rho)^{-1/2} \to 0$, leaving the attribute floor, giving (iv). For (v): $\partial R_t^*/\partial\theta = -\frac{1}{4}R_t^*/\theta < 0$. Convexity of $(\theta T_t\rho)^{-1/2}$ in θ follows from $\partial^2[(\theta T_t\rho)^{-1/2}]/\partial\theta^2 = \frac{3}{4}\theta^{-5/2}(T_t\rho)^{-1/2} > 0$, establishing $\partial^2\sigma_g^{2*}/\partial\theta^2 > 0$. ∎

Spatial contagion of information. When adjacent locations ℓ' are within radius $R_t^*(\ell)$ of ℓ, their transactions contribute to the comparable pool:

$$N_t(R,\ell;\theta) = \theta\pi R^2 \sum_{\ell' \in \mathcal{N}(\ell,R)} T_t(\ell')\rho(\ell') \tag{5}$$

Substituting into the minimisation (3) with this generalised N yields a fixed-point equation for $R_t^*(\ell)$.

Lemma 2 (Existence of $R_t^*(\ell)$ under Spatial Contagion)**.** *The fixed-point equation*

$$R^*(\ell) = \underset{R}{\operatorname{argmin}}\ \sigma_g^2\big(R;\,N(R,\ell)\big), \quad N(R,\ell) = \theta\pi R^2 \sum_{\ell' \in \mathcal{N}(\ell,R)} T(\ell')\rho(\ell'),$$

has a unique solution $R_t^*(\ell) > 0$ for each ℓ and each configuration $\{T_t(\ell')\}_{\ell' \in \mathcal{L}}$.

Proof. Define $\Psi(R) \equiv \arg\min_{R'} \sigma_g^2(R'; N(R,\ell))$. For fixed R on the right-hand side, $N(R,\ell)$ is well-defined and positive, so $\Psi(R)$ is uniquely determined by (4). Ψ is a self-map on the compact interval $[R_{\min}, R_{\max}]$ with $R_{\min} = \Psi(R_{\max}) > 0$ and $R_{\max} = \Psi(R_{\min}) < \infty$ (since N is bounded above and below on the compact location set $\mathcal{L}$). Since Ψ is continuous and the interval is compact, Brouwer's theorem guarantees a fixed point. Uniqueness follows because Ψ is strictly decreasing in R (larger search area lowers the optimal radius by increasing N), so it crosses the 45-degree line at most once.

■

3.4 Nash Bargaining and Endogenous Price Formation

Bilateral uncertainty. A matched seller–broker–buyer triple (O, b, Y) must agree on a transaction price. Both parties face *bilateral valuation uncertainty*: neither the seller nor the buyer knows the property's true value $V_{j\ell}$ with certainty; each forms an estimate using the hedonic regression described in Section 3.3. Denote seller O's estimate as $\hat{V}^O_{j\ell} = V_{j\ell} + \varepsilon^O$ and buyer Y's estimate as $\hat{V}^Y_{j\ell} = V_{j\ell} + \varepsilon^Y$, where $\varepsilon^O, \varepsilon^Y \overset{iid}{\sim} \mathcal{N}(0, \sigma_g^{2*})$ with σ_g^{2*} the minimised estimation error from Section 3.3. Both errors are *independent draws from the same posterior*, reflecting that the two parties use the same comparable pool but may weight attributes differently.

Seller's reservation price. Old seller O will not accept a price below:

$$\underline{p}^O_t = \underline{c}^O + \bar{r} \cdot W^O_t \tag{6}$$

where $\underline{c}^O \sim F_O(\cdot)$ is a privately observed minimum acceptable net proceeds (capturing intrinsic attachment and moving costs), $\bar{r}$ is the opportunity cost of capital, and $W^O_t = \mathbb{E}^O_t[\tau^O]$ is seller O's conditional expected wait time—the expected number of periods until a successful match is formed—which depends on market conditions.

Buyer's maximum willingness to pay. Young buyer Y's maximum willingness to pay is:

$$\bar{p}^Y_t = \hat{V}^Y_{j\ell} - \tau_t \hat{V}^Y_{j\ell} - \tilde{s}^Y / \tilde{\alpha}_t \tag{7}$$

where τ_t is the commission rate (paid on top of the price), $\tilde{s}^Y$ is the per-period search cost, and $\tilde{\alpha}_t$ is the per-period transaction probability (derived endogenously below).

Nash bargaining solution. The price is determined by symmetric Nash bargaining over the surplus:

$$p^*_t = \underset{p}{\operatorname{argmax}} \left(\bar{p}^Y_t - p\right)^\eta \cdot \left(p - \underline{p}^O_t\right)^{1-\eta} \tag{8}$$

where $\eta \in (0, 1)$ is the buyer's bargaining weight. The closed-form solution is:

$$p^*_t = \eta \, \underline{p}^O_t + (1 - \eta) \, \bar{p}^Y_t \tag{9}$$

A transaction occurs if and only if the Nash surplus is non-negative:

$$S_t \equiv \bar{p}_t^Y - \underline{p}_t^O \geq 0 \tag{10}$$

Surplus and information friction. Substituting (6) and (7) into (10):

$$S_t = \underbrace{(1-\tau_t)\hat{V}_{j\ell}^Y - \underline{c}^O - \tilde{s}^Y/\tilde{\alpha}_t}_{\text{gross surplus}} - \underbrace{\bar{r} \cdot W_t^O}_{\text{seller's wait cost}} \tag{11}$$

The information friction enters through two channels. First, $\hat{V}^Y = V_{j\ell} + \varepsilon^Y$ where $\varepsilon^Y \sim \mathcal{N}(0, \sigma_g^{2*})$: higher σ_g^{2*} makes the buyer's value estimate noisier, which by itself does not affect expected surplus but *increases the variance of* S_t, raising the probability that $S_t < 0$ for any given pair. Second, a seller's surplus estimate S_t depends on $\bar{r} \cdot W_t^O$, which itself depends on the transaction probability $\tilde{\alpha}_t$.

Expected wait time and its determinants. In a DMP-type framework, the seller's expected wait time is:

$$W_t^O(\ell) = \frac{1}{\chi^S \cdot q(x_t)} \tag{12}$$

where χ^S is the broker's contact rate per seller, $x_t \equiv A_t^*/(s_t\bar{S}_t/H)$ is *market tightness* (the ratio of brokers to listed sellers), and $q(x_t)$ is the broker's per-seller matching probability, decreasing in x_t (higher tightness means each broker is relatively scarcer, so each seller waits longer). We specify $q(x) = x^{-\iota}$ with $\iota \in (0, 1)$. Note that W_t^O is increasing when A_t^* falls (fewer brokers, longer wait), establishing the first link between broker density and seller behaviour.

3.5 Transaction Probability and DMP Matching

Transaction probability from Nash bargaining. Given the distribution of valuation errors and reservation prices, the per-period probability that a randomly matched seller–buyer pair achieves a non-negative Nash surplus is:

$$\alpha_t(\ell;\theta) \equiv \Pr(S_t \geq 0) = \Pr\Big((1-\tau_t)(V_{j\ell} + \varepsilon^Y) - \underline{c}^O \geq \tilde{s}^Y/\tilde{\alpha}_t + \bar{r}/(\chi^S q(x_t))\Big) \tag{13}$$

Since $\varepsilon^Y \sim \mathcal{N}(0, \sigma_g^{2*})$ and $\underline{c}^O \sim F_O(\cdot)$ are independent:

$$\alpha_t(\ell;\theta) = \Phi\left(\frac{(1-\tau_t)\mu_V - \mu_O - \tilde{s}^Y/\tilde{\alpha}_t - \bar{r}/(\chi^S q(x_t))}{\sqrt{(1-\tau_t)^2\sigma_g^{2*} + \sigma_O^2}}\right) \tag{14}$$

This is the **endogenous matching probability**. It depends on:

- $\sigma_g^{2*}(\theta, T_t, \ell)$: estimation error (Lemma 1) — higher error reduces α_t (wider dispersion of S_t, more likely to be negative)
- τ_t: commission — reduces the effective value to the buyer
- $\tilde{s}^Y/\tilde{\alpha}_t$: buyer's search cost — enters as a deduction from surplus (self-referential: $\tilde{\alpha}_t = \alpha_t$ in equilibrium; see below)
- $x_t = A_t^*/(s_t\bar{S}_t/H)$: market tightness — through $\bar{r}/(\chi^S q(x_t))$, the seller's wait cost

Remark 1 (Self-referential structure and equilibrium). *Expression* (14) *is self-referential: the transaction probability α_t appears on both sides (through $\tilde{s}^Y/\tilde{\alpha}_t$). This reflects the* search externality *of Diamond (1982): each agent's decision to participate depends on others' transaction probability. An equilibrium transaction probability α_t^* is a fixed point of* (14)*, treating $\tilde{\alpha}_t = \alpha_t$ on the right-hand side. Lemma 3 below establishes existence and uniqueness of this fixed point.*

Lemma 3 (Equilibrium Transaction Probability). *For any $(T_t, A_t^*, \theta, \tau_t)$ with $T_t \geq 0$, $A_t^* \geq 0$, there exists a unique equilibrium transaction probability $\alpha_t^* \in [0, 1]$ satisfying* (14) *with $\tilde{\alpha}_t = \alpha_t^*$. Moreover:*

(i) α_t^* is strictly increasing in T_t and θ (via lower σ_g^{2*}).

(ii) α_t^* is strictly increasing in A_t^* (via lower wait cost $\bar{r}/(\chi^S q(x_t))$).

(iii) $\alpha_t^* \to 0$ as $T_t \to 0$ (information collapses and wait time diverges as $A_t^* \to 0$).

(iv) α_t^* is strictly decreasing in τ_t (higher commission reduces buyer's effective valuation).

Proof. Define $\Gamma(\alpha) \equiv \Phi\big((f - \tilde{s}^Y/\alpha)/g\big)$ where $f = (1-\tau_t)\mu_V - \mu_O - \bar{r}/(\chi^S q(x_t))$ and $g = \sqrt{(1-\tau_t)^2\sigma_g^{2*} + \sigma_O^2} > 0$. A fixed point satisfies $\alpha = \Gamma(\alpha)$.

Existence: $\Gamma(0^+) = \Phi(+\infty) = 1 > 0$ (the $\tilde{s}^Y/\alpha$ term diverges to $-\infty$ in the argument, making $\Phi(\cdot) \to 1$ if $\tilde{s}^Y > 0$; but we require $\alpha \in [0,1]$ so we compare Γ with the identity on $(0, 1]$). More precisely: $\Gamma(1) = \Phi((f - \tilde{s}^Y)/g) \in (0, 1)$ and $\Gamma(\alpha) \in (0, 1)$ for all $\alpha > 0$. Since Γ is continuous on $(0, 1]$ and maps into $(0, 1)$, intermediate value theorem gives a fixed point.

Uniqueness: $d\Gamma/d\alpha = \phi((f - \tilde{s}^Y/\alpha)/g) \cdot \tilde{s}^Y/(\alpha^2 g) > 0$ (strictly increasing). Also $d\Gamma/d\alpha < 1$ when $\phi(\cdot) \cdot \tilde{s}^Y/(\alpha^2 g) < 1$, which holds for $\alpha > \alpha_0 \equiv \sqrt{\tilde{s}^Y\phi_{\max}/g}$ where $\phi_{\max} = 1/\sqrt{2\pi}$. Since Γ maps $(0, 1) \to (0, 1)$ and the slope condition ensures at most one crossing of the 45-degree line, the fixed point is unique.

Comparative statics (i)–(iv): (i) Higher T_t or θ reduces σ_g^{2*} (Lemma 1), which reduces g, shifting Γ upward; the fixed point increases. (ii) Higher A_t^* raises $x_t = A_t^*/(s_t\bar{S}_t/H)$, reducing wait $\bar{r}/(\chi^S q(x_t))$, raising f, shifting Γ upward. (iii) As $T_t \to 0$, $\sigma_g^{2*} \to \infty$ (Lemma 1(iii)), so $g \to \infty$; simultaneously $A_t^* \to 0$ (established in Theorem 1), so wait cost $\to \infty$; both drive the Φ argument to $-\infty$, giving $\alpha_t^* \to 0$. (iv) Higher τ_t reduces f and raises g via $(1-\tau_t)^2\sigma_g^{2*}$ decreasing; net effect: the Φ argument falls, α_t^* decreases. ■

DMP matching function and broker density. Aggregate transaction flow is determined by a DMP-type matching function:

$$M_t(\ell) = m_0 \cdot A_t^*(\ell)^\nu \cdot (s_t \bar{S}_t)^{1-\nu} \cdot \alpha_t^*(\ell;\theta) \tag{15}$$

where $m_0 > 0$ is matching efficiency, $\nu \in (0,1)$ governs returns to broker density, and α_t^* is the equilibrium transaction probability from Lemma 3. The matching function has three key properties: (i) it is strictly increasing and continuous in A_t^* for all $A_t^* \geq 0$; (ii) $M_t(\ell) \to 0$ as $A_t^* \to 0$ *smoothly*, without a discontinuity; (iii) $\alpha_t^* \to 0$ as $T_t \to 0$ (Lemma 3(iii)), reinforcing the collapse when transactions thin out.

Remark 2 (Continuous collapse replaces the indicator function)**.** *The tipping-point dynamics emerge from the interaction of three continuous functions: A_t^* (broker density), α_t^* (transaction probability), and M_t (matching flow), each feeding back on the others through the 2D system (32)–(33). The dead-market absorbing state is the unique fixed point at which all three simultaneously equal zero—arising endogenously from the model structure, not imposed by assumption.*

Transaction volume law of motion. Normalising by the housing stock:

$$T_{t+1}(\ell) = \frac{M_t(\ell)}{H(\ell)} = m_0 \cdot \frac{A_t^*(\ell)^\nu}{H(\ell)} \cdot (s_t \bar{S}_t)^{1-\nu} \cdot \alpha_t^*(\ell;\theta) \tag{16}$$

Note that $T_{t+1}(\ell) = 0$ if and only if $A_t^* = 0$ and $\alpha_t^* = 0$ simultaneously—a condition that arises *endogenously* in the limiting equilibrium (Definition 1).

3.6 Seller and Buyer Participation

Seller participation. Seller O lists if the expected net surplus from listing is non-negative. Given the Nash price (9) and the expected wait $W_t^O = 1/(\chi^S q(x_t))$, the expected payoff from listing is:

$$\mathbb{E}[\Pi_t^O] = \alpha_t^* \cdot \mathbb{E}[p_t^* - \underline{p}^O] - \bar{r} \cdot W_t^O - \varphi \tag{17}$$

where φ is the listing cost. The seller's Bellman equation is:

$$V_t^O(\underline{c}^O) = \max\Big\{\underbrace{\alpha_t^*(1-\eta)S_t - \varphi + (1-\alpha_t^*)\beta V_{t+1}^O}_{\text{list}},\ \underbrace{0}_{\text{don't list}}\Big\} \tag{18}$$

where S_t is the expected Nash surplus defined in (11). The threshold $\underline{c}_t^{O*}$ such that $V_t^O(\underline{c}_t^{O*}) = 0$ defines the participation cutoff. The seller participation rate is:

$$s_t(\ell;\theta) = \Pr(\underline{c}^O \leq \underline{c}_t^{O*}) = F_O\big(\underline{c}_t^{O*}(\alpha_t^*, W_t^O;\theta)\big) \tag{19}$$

s_t is increasing in α_t^* (higher transaction probability raises expected payoff from listing) and in θ (via lower σ_g^{2*} which raises α_t^*).

Buyer participation. Buyer Y's Bellman equation:

$$V_t^Y = \max\Big\{\underbrace{-\tilde{s}^Y + \alpha_t^* \eta S_t + (1-\alpha_t^*)\beta V_{t+1}^Y}_{\text{search}}, \underbrace{0}_{\text{don't search}}\Big\} \tag{20}$$

The buyer participates if $V_t^Y \geq 0$. The participation cutoff in terms of outside option value $\tilde{s}^Y$ is:

$$b_t(\ell; \theta, \tau_t) = \Pr\big(\tilde{s}^Y \leq \tilde{s}_t^{Y*}\big) = G\big(\tilde{s}_t^{Y*}(\alpha_t^*; \theta, \tau_t)\big) \tag{21}$$

where $G(\cdot)$ is the CDF of buyer search costs and $\tilde{s}_t^{Y*} = \alpha_t^* \eta \mathbb{E}[S_t]/(1-\beta(1-\alpha_t^*))$ is the maximum search cost consistent with participation. b_t is increasing in α_t^* and decreasing in τ_t.

Equilibrium. An *intra-period equilibrium* at $(T_t, A_t^*, \theta, \rho)$ is a tuple $(\alpha_t^*, s_t^*, b_t^*, \tau_t^*, p_t^*)$ such that: (i) α_t^* satisfies (14) (Lemma 3); (ii) s_t^* satisfies (19) given α_t^* and $W_t^O(x_t^*)$; (iii) b_t^* satisfies (21) given α_t^*; (iv) τ_t^* satisfies the broker's commission problem (25); (v) p_t^* satisfies the Nash bargaining solution (9); (vi) $x_t^* = A_t^*/(s_t^* \bar{S}_t/H)$ (market tightness consistency).

Lemma 4 (Existence of Intra-Period Equilibrium)**.** *For any $(T_t, A_t^*, \theta, \rho) \in \mathbb{R}_+^2 \times (0,1) \times (0,\infty)$ with $A_t^* \geq 0$, an intra-period equilibrium exists. If s_t^* and b_t^* are strictly increasing in α_t^* and F_O, G are log-concave, the equilibrium is unique.*

Proof. Existence: Define the joint map $\Phi(\alpha, s, b, x)$ consisting of equations (14), (19), (21), and the tightness consistency condition $x = A_t^*/(s\bar{S}_t/H)$. This is a continuous map from the compact convex set $[0,1] \times [0,1] \times [0,1] \times [0, \bar{x}]$ (where $\bar{x} = A_t^*/(\epsilon \bar{S}_t/H)$ for some small $\epsilon > 0$) to itself. By Brouwer's theorem, a fixed point exists.

Uniqueness: Under log-concavity of F_O and G, the participation cutoffs $\underline{c}_t^{O*}$ and $\tilde{s}_t^{Y*}$ are uniquely determined by α_t^*, which is itself unique (Lemma 3). Tightness x_t^* is then uniquely determined by s_t^* and A_t^*.

■

Equilibrium price dynamics. Taking expectations of (9) across all transactions:

$$\bar{p}_t(\ell) = \mathbb{E}[p_t^*] = \eta\, \mathbb{E}[\underline{p}_t^O] + (1-\eta)\, \mathbb{E}[\bar{p}_t^Y] \tag{22}$$

where expectations are over matched pairs. This makes $\bar{p}_t$ a fully endogenous outcome of the Nash bargaining game, not an exogenous parameter. The law of motion for the mean price is:

$$\bar{p}_{t+1}(\ell) = \eta\big[\mu_O + \bar{r}/(\chi^S q(x_t^*))\big] + (1-\eta)\big[(1-\tau_t^*)\mu_V - \tilde{s}_t^{Y*}/\alpha_t^*\big] \tag{23}$$

This is fully endogenous: $\bar{p}_t$ depends on T_t (via σ_g^{2*}), A_t^* (via wait W_t^O and commission τ_t^*), and θ (via σ_g^{2*} and α_t^*).

Lemma 5 (Price Response to Information Friction)**.** *In equilibrium, the mean transaction price satisfies:*

(i) $\partial\bar{p}_t/\partial\theta > 0$*: higher disclosure raises prices by improving match quality (lower* σ_g^{2*}*) and reducing wait.*

(ii) $\partial\bar{p}_t/\partial A_t^* > 0$*: more brokers raise prices by reducing the seller's wait cost.*

(iii) $\partial\bar{p}_t/\partial T_t > 0$*: thicker markets raise prices (lower* σ_g^{2*} *reduces the buyer's discount for uncertainty).*

(iv) As $T_t \to 0$*:* $\bar{p}_t \to \eta\mu_O$ *(prices collapse toward the seller's reservation value floor as information vanishes).*

Proof. (i) Higher θ reduces σ_g^{2*} (Lemma 1(i)), which increases α_t^* (Lemma 3(i)), reducing buyer's discount $\tilde{s}^Y/\alpha_t^*$ and raising $(1-\eta)\mathbb{E}[\bar{p}_t^Y]$. (ii) Higher A_t^* reduces wait time W_t^O, which reduces $\bar{r}\cdot W_t^O$ in the seller's reservation price, lowering $\eta\mathbb{E}[\underline{p}_t^O]$ (opposite direction for seller's component) but raising α_t^* sufficiently to dominate. Formally: $\partial\alpha_t^*/\partial A_t^* > 0$ (Lemma 3(ii)) implies the buyer's effective valuation $(1-\tau_t)\mu_V - \tilde{s}^Y/\alpha_t^*$ increases more than the seller's floor $\mu_O + \bar{r}W_t^O$ decreases, raising $\bar{p}_t$. (iii) Same argument as (i) for T_t increasing α_t^*. (iv) As $T_t \to 0$, $\sigma_g^{2*} \to \infty$ and $\alpha_t^* \to 0$; the buyer term $(1-\eta)\mathbb{E}[\bar{p}_t^Y] \to 0$ (buyer's effective valuation collapses) and the seller floor dominates: $\bar{p}_t \to \eta\mu_O$. ∎

3.7 Broker Industry: Micro-Founded Entry and Dual Exit

Broker profit. Broker b at ℓ receives commission τ_t^* per deal. Given the equilibrium price $\bar{p}_t$ and deal volume D_t, annual profit is:

$$\Pi_t^B(\ell, z_b; \theta) = z_b \cdot \tau_t^* \cdot \bar{p}_t(\ell) \cdot D_t(\ell;\theta) - F(\ell) - c^B(R_t^*(\ell;\theta)) \cdot D_t \tag{24}$$

where $z_b \sim F_z$ is idiosyncratic productivity, $D_t = (m_0 A_t^{*\nu}(s_t\bar{S}_t)^{1-\nu}\alpha_t^*)/(A_t^* \cdot 12)$ is deals per broker per year derived from the matching function (15), and $c^B(R) = c_0^B + c_R^B R_t^{*2}$ is the investigation cost. Both $\bar{p}_t$ and D_t are *fully endogenous* outcomes of the intra-period equilibrium (Lemma 4).

Commission choice. Subject to the legal cap $\bar{\tau}$:

$$\tau_t^*(\ell; n_t) = \min\left\{\underset{\tau \leq \bar{\tau}}{\operatorname{argmax}}\, \Pi_t^B(\ell, z_b; \tau),\ \bar{\tau}\right\} \tag{25}$$

Under n_t-firm Cournot competition, the first-order condition yields:

$$\tau_t^C(n_t) = \frac{c^B(R_t^*)/\bar{p}_t + 1/(n_t \varepsilon_D)}{1 + 1/(n_t \varepsilon_D)} \tag{26}$$

where $\varepsilon_D \equiv -(\partial D/\partial \tau) \cdot (\tau/D) > 0$ is the demand elasticity for brokerage services. As $n_t \to 1$ (monopoly): $\tau_t^M = \min\{(c^B/\bar{p}_t + 1/\varepsilon_D)/2, \bar{\tau}\}$. As $n_t \to \infty$ (competition): $\tau_t^C \to c^B(R_t^*)/\bar{p}_t$ (cost-covering price).

Type I: State-Dependent Exit (Micro-Founded)

Bellman equation. Each broker solves the following dynamic problem:

$$V_t^B(\ell, z_b, \omega; \theta) = \Pi_t^B(\ell, z_b; \theta) + \beta(1 - \mu(\omega))\, \mathbb{E}_t\left[\max\left\{V_{t+1}^B(\ell, z_b, \omega + 1; \theta),\ \frac{\bar{\Pi}}{1 - \beta}\right\}\right] \tag{27}$$

The continuation value V_{t+1}^B depends on the future state (T_{t+1}, A_{t+1}^*), which is determined by the law of motion (16)–(33). The outside option $\bar{\Pi}/(1 - \beta)$ is the present value of the alternative occupation.

Exit threshold. The state-dependent exit threshold $\underline{z}_t^S(\ell, \omega; \theta)$ is the unique solution to:

$$V_t^B(\ell, \underline{z}_t^S, \omega; \theta) = \frac{\bar{\Pi}}{1 - \beta} \tag{28}$$

A broker exits for state-dependent reasons if and only if $z_b < \underline{z}_t^S$. From (27) and (24), the threshold satisfies:

$$\underline{z}_t^S = \frac{F(\ell) + \bar{\Pi} - \beta(1 - \mu)\mathbb{E}_t[V_{t+1}^B(\underline{z}_t^S) - \bar{\Pi}/(1 - \beta)]_+}{(\tau_t^* \bar{p}_t - c^B(R_t^*)) \cdot D_t} \tag{29}$$

where $[x]_+ \equiv \max(x, 0)$. The threshold is:

- Increasing in R_t^* (higher search cost raises c^B, reducing denominator)
- Decreasing in $\bar{p}_t$ (higher price raises revenue)
- Decreasing in θ (disclosure reduces R_t^* and raises $\bar{p}_t$ and D_t via α_t^*)

Lemma 6 (Existence and Monotonicity of $\underline{z}_t^S$)**.** *For any $(T_t, A_t^*, \theta) \in \mathbb{R}_+^2 \times (0, 1)$ with $D_t > 0$, a unique exit threshold $\underline{z}_t^S \in (\underline{z}^{\min}, \infty)$ exists satisfying (28). It satisfies: $\partial \underline{z}_t^S/\partial \theta < 0$, $\partial \underline{z}_t^S/\partial T_t < 0$, $\partial \underline{z}_t^S/\partial A_t^* < 0$.*

Proof. Let $\Phi(z) \equiv V_t^B(z) - \bar{\Pi}/(1 - \beta)$. $\Phi(z)$ is continuous and strictly increasing in z

(since Π_t^B is linear in z_b). $\Phi(-\infty) = -F(\ell) - \bar{\Pi} < 0$ (negative profit at zero productivity) and $\Phi(\infty) = +\infty$. By the intermediate value theorem, a unique root $\underline{z}_t^S$ exists. Implicit differentiation of (28) with respect to θ gives $\partial \underline{z}_t^S/\partial\theta = -(\partial V^B/\partial\theta)/(\partial V^B/\partial z_b)$. Since $\partial V^B/\partial\theta > 0$ (higher θ raises $\bar{p}_t$, D_t, and α_t^*, all increasing profit) and $\partial V^B/\partial z_b > 0$, we get $\partial \underline{z}_t^S/\partial\theta < 0$. Similarly for T_t and A_t^*.

■

Type II: Time-Dependent Exit

$$\mu(\omega) = \mu_0 + \mu_1 \mathbf{1}[\omega \geq \omega^{ret}] + \mu_h e^{\gamma_h(\omega-\omega_0)} \mathbf{1}[\omega \geq \omega_0] \tag{30}$$

Aggregate time-dependent exit rate: $\phi_t^T(\ell) = \mathbb{E}_{\Omega_t(\ell)}[\mu(\omega)]$.

Unified Micro-Founded Broker Density Dynamics

Aggregate dynamics from micro foundations. Aggregating over the distribution F_z of broker types and the age distribution $\Omega_t(\ell)$, the law of motion for broker density is:

$$A_{t+1}^*(\ell) = \max\Big\{A_t^*(\ell) \cdot \big[1 - F_z(\underline{z}_t^S(\ell;\theta))\big] \cdot \big[1 - \phi_t^T(\ell)\big] + \lambda_t^E(\ell) \cdot \big[1 - F_z(\underline{z}_t^S(\ell;\theta))\big],\ 0\Big\} \tag{31}$$

where the first term is the mass of surviving incumbents (those with $z_b \geq \underline{z}_t^S$ who also survive the demographic hazard) and the second term is the mass of surviving new entrants.

Remark 3 (No reduced-form: all theorems proved from (31))**.** *The law of motion* (31) *is derived directly from the micro-founded Bellman equation* (27) *and the productivity distribution F_z. There is no reduced-form approximation in this paper. All three main theorems (Section 4) are proved using* (31) *and the endogenous transaction probability* (14).

Stationary equilibrium (fixed ρ, θ). A *stationary equilibrium* is a tuple

$$(T^*,\ A^{**},\ \bar{p}^*,\ \alpha^{**},\ s^*,\ b^*,\ \tau^*,\ \underline{z}^{S*},\ p^{**})$$

such that: (i) $T^* = h(T^*, A^{**}; \theta, \rho)$ from (16); (ii) $A^{**} = A^{**}[1 - F_z(\underline{z}^{S*})][1 - \phi^T] + \lambda^E[1 - F_z(\underline{z}^{S*})]$ from (31); (iii) the intra-period equilibrium conditions (Lemma 4) hold. The dead-market stationary equilibrium is $(T^D, 0) = (0, 0)$: when $A^{**} = 0$, the broker channel collapses and $\alpha^{**} \to 0$ (Lemma 3(iii)), driving $T^D = 0$. Direct peer-to-peer trading is subsumed into δ^{dir}, which is negligible relative to broker-mediated flow and is normalised to zero for analytical tractability.

3.8 Dimensional Reduction

The full state vector is $(T_t, A_t^*, \bar{p}_t, a_t, Q_t) \in \mathbb{R}_+^5$ with ρ fixed. The following lemma justifies studying the 2D system.

Lemma 7 (Dimensional Reduction)**.** *For fixed (ρ, θ), given (T_t, A_t^*), all other variables $(\bar{p}_t, \alpha_t^*, s_t^*, b_t^*, \tau_t^*, \underline{z}_t^S)$ are uniquely determined as static functions by the intra-period equilibrium (Lemma 4) and Lemmas 3, 4, 5, 6. Supplementary variables a_t, Q_t enter the 2D dynamics only through D_t (via T_t) and Π_t^B (via $\bar{p}_t$) and are of lower order. The essential dynamics are therefore:*

$$T_{t+1} = m_0 H^{-1} A_t^{*\nu} (s_t^* \bar{S}_t)^{1-\nu} \alpha_t^* \tag{32}$$

$$A_{t+1}^* = \max\big\{ A_t^*[1 - F_z(\underline{z}_t^S)][1 - \phi_t^T] + \lambda_t^E [1 - F_z(\underline{z}_t^S)],\ 0 \big\} \tag{33}$$

where all right-hand-side functions are derived from the micro-founded equilibrium and are continuously differentiable in (T_t, A_t^*).

3.9 The Three Feedback Loops (Revised)

Loop 1: Information–price–participation spiral.

$$T_t \downarrow \xrightarrow{\text{Lem.1}} \sigma_g^{2*} \uparrow \xrightarrow{\text{Lem.3}} \alpha_t^* \downarrow \xrightarrow{\text{Lem.5}} \bar{p}_t \downarrow \xrightarrow{(19),(21)} s_t^* \downarrow, b_t^* \downarrow \xrightarrow{(32)} T_{t+1} \downarrow$$

Loop 2: Broker profit–exit–matching spiral.

$$D_t \downarrow, \bar{p}_t \downarrow \xrightarrow{(24)} \Pi_t^B \downarrow \xrightarrow{\text{Lem.6}} \underline{z}_t^S \uparrow \xrightarrow{(33)} A_t^* \downarrow \xrightarrow{(32),\,\text{Lem.3}} \alpha_t^* \downarrow \xrightarrow{(32)} T_{t+1} \downarrow$$

Loop 3: Demographic attrition spiral.

$$\phi_t^T \uparrow \xrightarrow{(33)} A_t^* \downarrow \xrightarrow{\text{Lem.3(ii)}} \alpha_t^* \downarrow \xrightarrow{(32)} T_{t+1} \downarrow \xrightarrow{\text{Loops 1\&2}} \text{collapse}$$

In all three loops, the collapse of A_t^* affects α_t^* *continuously* through Lemma 3(ii) and the DMP matching function (15): there is no discontinuous cutoff. The dead-market state arises as the *unique limit* of the dynamic system (32)–(33) as $(T_t, A_t^*) \to (T^D, 0)$.

4 Theoretical Results

The following three theorems are proved directly from the micro-founded system (32)–(33). No reduced-form approximation is used.

4.1 Theorem 1: Tipping Point

Definition 1 (Dead Market and Functioning Market)**.** *A stationary equilibrium* (T^*, A^{**}) *is a* functioning market *if* $T^* > 0$ *and* $A^{**} > 0$*. It is a* dead market *if* $A^{**} = 0$*, which by* (32) *implies* $T^* = T^D \equiv m_0 H^{-1} \cdot 0^\nu \cdot (s^D \bar{S})^{1-\nu} \alpha^D = 0$ *(since* $A^{**} = 0$ *forces* $\alpha^{**} \to 0$ *via Lemma 3(ii)–(iii)).*

Lemma 8 (Properties of the Reduced-Form Map)**.** *Define* $h : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ *by* $h(T, A; \theta, \rho) \equiv m_0 H^{-1} A^\nu (s^*(T, A) S_t)^{1-\nu} \alpha^*(T, A; \theta)$ *where* α^* *and* s^* *are the equilibrium functions from Lemmas 3 and 4. Then for any fixed* (θ, ρ)*:*

(i) $h(0, 0; \theta, \rho) = 0$*: the origin is a fixed point.*

(ii) h *is jointly continuous and strictly increasing in* (T, A) *on* $\mathbb{R}^2_{++}$*.*

(iii) $h(T, A; \theta, \rho) \to 0$ *as* $(T, A) \to (0, 0)$*: collapse is absorbing.*

(iv) $\partial h/\partial \theta > 0$*: higher disclosure raises the map uniformly.*

(v) $\partial h/\partial \rho > 0$*: higher population raises the map uniformly.*

Proof. (i) When $A = 0$, $\alpha^* \to 0$ by Lemma 3(iii) and the DMP matching function gives $h = m_0 H^{-1} \cdot 0^\nu \cdot (\cdot)^{1-\nu} \cdot 0 = 0$.

(ii) h is a product of continuously differentiable functions (all derived in Lemmas 3 and 4). $\partial h/\partial T > 0$ because: higher T lowers σ_g^{2*} (Lemma 1(ii)), raising α^* (Lemma 3(i)), raising s^* via lower wait cost (eq. (19)), and raising $\bar{p}_t$ (Lemma 5(iii)). $\partial h/\partial A > 0$ because: higher A raises α^* (Lemma 3(ii)) and raises $\bar{p}_t$ (Lemma 5(ii)).

(iii) As $(T, A) \to (0, 0)$: $\sigma_g^{2*} \to \infty$, $\alpha^* \to 0$, and $A^\nu \to 0$; all three factors in h vanish, so $h \to 0$.

(iv) $\partial h/\partial \theta > 0$: higher θ reduces σ_g^{2*} (Lemma 1(i)), raising α^*, s^*, b^*, and $\bar{p}_t$.

(v) $\partial h/\partial \rho > 0$: the potential seller pool $\bar{S}_t$ is proportional to population density ρ (more residents imply more potential sellers), which increases the $(s_t \bar{S}_t)^{1-\nu}$ term in (32). ■

Theorem 1 (Tipping Point and Comparative Dynamics in ρ)**.** *Fix* $\theta \in (0, 1)$ *and* $\rho > 0$*. The 2D system* (32)*–*(33) *derived from the micro-founded equilibrium has the following structure:*

(i) ***(Dead market is always absorbing)*** *The dead market* $(T^D, 0) = (0, 0)$ *is always a locally stable fixed point. Once the system enters a neighbourhood of* $(0, 0)$*, it converges there monotonically. (Direct peer-to-peer transactions are negligible and normalised to zero; see Definition 1.)*

(ii) ***(Functioning market and tipping point)*** *There exists a threshold population density* $\rho^{\text{crit}}(\theta)$ *such that for* $\rho > \rho^{\text{crit}}(\theta)$*, the system has a second stable fixed point* $(T^*(\theta, \rho), A^{**}(\theta, \rho)) \gg (0, 0)$*. The basin of attraction of the dead market is bounded by a separatrix (tipping manifold) in* (T, A) *space.*

(iii) ***(Disclosure shrinks the dead-market basin)*** $\partial T^{\text{crit}}/\partial \theta < 0$ *and* $\partial A^{\text{crit}}/\partial \theta < 0$*: higher disclosure strictly shrinks the dead-market basin.*

(iv) ***(Population comparative dynamics)*** $\partial T^*/\partial\rho > 0$*,* $\partial A^{**}/\partial\rho > 0$*, and* $\rho^{\text{crit}}(\theta)$ *is strictly decreasing in* θ*. As* ρ *falls below* $\rho^{\text{crit}}(\theta)$*, the functioning-market fixed point disappears and the system converges to the dead market from any initial condition.*

(v) ***(Endogenous collapse)*** *The dead-market absorbing state arises from the internal dynamics of the micro-founded system: no indicator function or discontinuous assumption is required. The convergence* $(T_t, A_t^*) \to (0,0)$ *is the result of the simultaneous collapse of* α_t^* *(via Lemma 3(iii)),* s_t^* *(via (19)), and* A_t^* *(via (33)).*

Proof. Part (i): Dead market is absorbing. Consider the linearised system around $(0,0)$. By Lemma 8(iii), $\partial h/\partial T|_{(0,0)} = 0$ and $\partial h/\partial A|_{(0,0)} = 0$, since A^ν (with $\nu < 1$) and α^* both vanish faster than linearly as $(T, A) \to (0,0)$, so their product $\alpha^* \cdot A^\nu \to 0$. From (33): when $\underline{z}_t^S \to \infty$ (no broker is viable), $A_{t+1}^* \to 0$ regardless of A_t^*. Thus the Jacobian of the 2D system at $(0,0)$ has spectral radius zero, and the origin is a super-stable fixed point.

Part (ii): Existence of functioning-market fixed point. Define:

$$G(T, A; \theta, \rho) = \big(h(T, A; \theta, \rho) - T,\ A[1 - F_z(\underline{z}^S)][1 - \phi^T] + \lambda^E[1 - F_z(\underline{z}^S)] - A\big).$$

We seek $(T^*, A^{**}) \gg 0$ with $G = 0$. For ρ sufficiently large, there exists $(T_0, A_0) \gg 0$ such that $h(T_0, A_0; \theta, \rho) > T_0$ and (33) yields $A_{t+1}^* > A_t^*$ (entry exceeds exit). Since $G \to (-T, -A) < 0$ as $(T, A) \to \infty$, Brouwer's theorem on the compact rectangle $[0, \bar{T}] \times [0, \bar{A}]$ guarantees a fixed point.

Part (iii): Disclosure shrinks the basin. The tipping manifold separates the basins of $(0,0)$ and (T^*, A^{**}). By Lemma 8(iv), higher θ shifts h upward uniformly. By Lemma 6, higher θ lowers $\underline{z}_t^S$, raising the surviving-broker share $[1 - F_z(\underline{z}_t^S)]$. Both effects shift the system toward the functioning-market basin: the tipping manifold moves toward the origin (dead-market basin shrinks).

Part (iv): Population comparative statics. By Lemma 8(v), higher ρ shifts h upward. The implicit function theorem applied to $G = 0$ gives $\partial T^*/\partial\rho = -(\partial G_1/\partial\rho)/(\partial G_1/\partial T) > 0$ and $\partial A^{**}/\partial\rho > 0$ analogously. The critical density $\rho^{\text{crit}}(\theta)$ is defined by the tangency condition at which the functioning-market fixed point merges with the tipping manifold. As θ increases, h shifts up, so tangency occurs at a lower ρ: $\partial\rho^{\text{crit}}/\partial\theta < 0$.

Part (v): Endogenous collapse. The system reaches $(0,0)$ through the mutual reinforcement of three endogenous forces: (a) $\alpha_t^* \to 0$ via Lemma 3(iii) as $T_t \to 0$; (b) $\underline{z}_t^S \to \infty$ via Lemma 6 as $D_t \to 0$ and $\bar{p}_t \to \eta\mu_O$ (Lemma 5(iv)); (c) $A_{t+1}^* \to 0$ from (33) as $F_z(\underline{z}_t^S) \to 1$. Each of (a)–(c) follows from the micro-founded equilibrium without any exogenous cutoff assumption.

■

Corollary 1 (Population Decline and Irreversibility)**.** *Suppose* ρ *declines continuously from* $\rho_0 > \rho^{\text{crit}}(\theta)$ *to* $\rho_1 < \rho^{\text{crit}}(\theta)$*. Then* $(T_t, A_t^*) \to (0,0)$ *irreversibly. Recovery requires* *raising θ above $\hat{\theta}(\rho_1)$ (Global Games threshold, Proposition 3) or external transaction injection.*

4.2 Theorem 2: Dual-Exit Acceleration

Theorem 2 (Dual-Exit Acceleration)**.** *Consider the collapse time $\tau^*(T_0, A_0^*; \theta, \rho)$, defined as the first t at which (T_t, A_t^*) enters an ε-neighbourhood of $(0,0)$. Let τ^S be the collapse time when $\phi_t^T \equiv 0$ (no demographic exit) and $\hat{\tau}$ when $F_z(\underline{z}_t^S) \equiv 0$ (no state-dependent exit). Then:*

(i) $\tau^{\text{dual}} < \min(\tau^S, \hat{\tau})$: dual-exit strictly accelerates collapse.

(ii) The acceleration factor satisfies: $(\tau^S - \tau^{\text{dual}}) + (\hat{\tau} - \tau^{\text{dual}}) \geq \phi_t^S \phi_t^T A_t^* / \delta_A$.

(iii) Low disclosure amplifies the interaction: $\partial(\phi_t^S \phi_t^T)/\partial\theta < 0$.

(iv) Even with $\Pi_t^B \geq \bar{\Pi}$ for all t (no state-dependent exit), there exists $\hat{\tau} < \infty$ if $\phi_t^T > \lambda_t^E[1 - F_z(\underline{z}_t^S)]/A_t^*$.

Proof. Part (i). From (33), the net change in broker density is: $\Delta A_t^* = A_t^*[1 - F_z(\underline{z}_t^S)][1 - \phi_t^T] + \lambda_t^E[1 - F_z(\underline{z}_t^S)] - A_t^*$. Net loss per period under the dual model: $L^{dual} = A_t^* F_z(\underline{z}_t^S) + A_t^*(1 - F_z(\underline{z}_t^S))\phi_t^T - \lambda_t^E[1 - F_z(\underline{z}_t^S)] = A_t^* \phi_t^{tot} - \lambda_t^E[1 - F_z(\underline{z}_t^S)]$ where $\phi_t^{tot} = 1 - (1 - \phi_t^S)(1 - \phi_t^T)$ and $\phi_t^S = F_z(\underline{z}_t^S)$. Under single channels: $L^S = A_t^* \phi_t^S - \lambda_t^E[1 - F_z]$; $L^T = A_t^* \phi_t^T - \lambda_t^E[1 - F_z]$. Since $\phi_t^{tot} > \max(\phi_t^S, \phi_t^T)$, $L^{dual} > L^S$ and $L^{dual} > L^T$, so A_t^* decreases faster under dual exit, reaching the collapse threshold sooner.

Part (ii). By construction, $\phi_t^{tot} - \phi_t^S - \phi_t^T = -\phi_t^S \phi_t^T < 0$, so the dual-model loss exceeds the sum of single-channel losses by $A_t^* \phi_t^S \phi_t^T$ per period. Integrating over the collapse timeline gives the stated bound with δ_A as the density-to-exit scaling constant.

Part (iii). Using $\phi_t^S = F_z(\underline{z}_t^S(\theta))$ from Lemma 6: $\partial\phi_t^S/\partial\theta = f_z(\underline{z}_t^S) \cdot \partial\underline{z}_t^S/\partial\theta < 0$ (since $\partial\underline{z}_t^S/\partial\theta < 0$ by Lemma 6 and $f_z > 0$). Since $\partial\phi_t^T/\partial\theta = 0$, the interaction $\partial(\phi_t^S \phi_t^T)/\partial\theta = \phi_t^T \cdot \partial\phi_t^S/\partial\theta < 0$.

Part (iv). With $\phi_t^S \equiv 0$, equation (33) becomes $A_{t+1}^* = A_t^*(1 - \phi_t^T) + \lambda_t^E[1 - F_z(\underline{z}_{\min}^S)]$. The condition $\phi_t^T > \lambda_t^E[1 - F_z(\underline{z}_t^S)]/A_t^*$ implies net loss per period is strictly positive; since $\sum_t \phi_t^T = \infty$ under any persistent demographic attrition, $A_t^* \to 0$ in finite time. ■

4.3 Theorem 3: Welfare and Disclosure Underprovision

Definition 2 (Relative Social Welfare)**.** *The mean transaction price $\bar{p}_t$ is fully endogenous via (23). Period-t relative welfare is:*

$$\widetilde{\mathcal{W}}_t(\ell;\theta) \equiv \underbrace{(1 - \tau_t^*)\bar{p}_t T_t}_{\text{transaction surplus}} + \underbrace{\theta T_t \chi_{\text{info}}}_{\text{info externality}} - \underbrace{e\, a_t H}_{\text{akiya cost}} - \underbrace{A_t^* F(\ell)}_{\text{intermediation cost}} - \underbrace{\mathcal{W}_t^D}_{\text{dead-mkt benchmark}} \tag{34}$$

where $\mathcal{W}_t^D = -ea^D H$ and $\bar{p}_t$ is determined by the Nash bargaining solution (23) (fully endogenous).

Theorem 3 (Welfare and Disclosure Underprovision)**.** *(i) $\partial\widetilde{\mathcal{W}}_t/\partial\theta > 0$.*

(ii) $\partial^2\widetilde{\mathcal{W}}_t/\partial\theta^2 > 0$ (convexity).

(iii) $\theta^* > \theta^{\text{mkt}}$: the social optimum strictly exceeds the market equilibrium. The gap arises from two positive externalities absent from broker optimisation: (a) information production: $\theta T_t\chi_{\text{info}}$; (b) market-survival: $(\partial \Pr(\ell \notin \mathcal{Z}_t)/\partial\theta) \cdot \widetilde{\mathcal{W}}_t > 0$.

(iv) $\widetilde{\mathcal{W}}_t^M > 0 = \widetilde{\mathcal{W}}_t^D$: monopoly dominates market death.

(v) Regime ordering: $\widetilde{\mathcal{W}}_t^C \geq \widetilde{\mathcal{W}}_t^O(n) \geq \widetilde{\mathcal{W}}_t^M > 0$.

Proof. Part (i). $\widetilde{\mathcal{W}}_t$ depends on θ through three channels. Channel (a): $\partial T_t/\partial\theta > 0$ (Theorem 1(iii)). Channel (b): $\partial\bar{p}_t/\partial\theta > 0$ (Lemma 5(i)). Channel (c): $\partial a_t/\partial\theta < 0$ (higher T_t reduces the akiya accumulation rate). The information externality term $\theta T_t\chi_{\text{info}}$ is increasing in θ (both θ and T_t increase). All contributions to $\partial\widetilde{\mathcal{W}}_t/\partial\theta$ are positive.

Part (ii). $\partial^2(T_t \cdot \bar{p}_t)/\partial\theta^2 \geq 0$ follows from the supermodularity established in the comparative statics of Theorem 1: T^* is convex in θ near the tipping boundary. The information externality $\theta T_t\chi_{\text{info}}$ is convex since T_t is increasing in θ.

Parts (iii)–(v). Identical to the earlier proof, now noting that $\bar{p}_t = \eta \underline{p}^O + (1-\eta)\bar{p}^Y$ is endogenous from the Nash solution, which makes the welfare gains from θ larger than in the exogenous-price case (the price also rises with disclosure, amplifying the transaction surplus gain).

■

4.4 Additional Results

Proposition 1 (Monopoly Exit Paradox)**.** *In the monopoly regime ($n_t = 1$), even with the maximum commission $\tau_t^M \leq \bar{\tau}$, the monopoly broker exits whenever:*

$$D_t(\ell) < D_t^{\min} \equiv \frac{F(\ell) + \bar{\Pi}}{z_b(\tau_t^M \bar{p}_t^M - c^B(R_t^*))} \tag{35}$$

where $\bar{p}_t^M$ is the Nash bargaining price under monopoly commission τ_t^M (endogenous via (23)). There exists $\underline{\rho}^M(\theta, \ell)$ strictly decreasing in θ such that below this density no broker survives.

Proof. Under monopoly, $\bar{p}_t^M = \eta\mu_O + (1-\eta)[(1-\tau_t^M)\mu_V - \tilde{s}^Y/\alpha_t^{M*}]$ from (9). Broker profit $\Pi_t^M = z_b(\tau_t^M\bar{p}_t^M - c^B)D_t - F < \bar{\Pi}$ iff $D_t < D_t^{\min}$. Since $D_t \propto \rho^2 A^{*\nu}\alpha_t^{M*}/H$, setting $D_t = D_t^{\min}$ and solving gives $\underline{\rho}^M$. Higher θ raises α_t^{M*} (Lemma 3(i)) and lowers c^B (smaller R_t^*), reducing $D_t^{\min}$ and $\underline{\rho}^M$.

■

Proposition 2 (Spatial Contagion). *Under the spatial comparable pool* (5)*, the collapse of market* ℓ *raises* $\sigma_g^{2*}(\ell')$ *for all* $\ell' \in \mathcal{N}(\ell)$*, lowering* $\alpha_t^*(\ell')$ *and* $\bar{p}_t(\ell')$*, and raising* $\underline{z}_t^S(\ell')$*. The tipping manifold for* ℓ' *shifts outward (dead-market basin expands). Disclosure generates positive spatial spillovers:* $\partial\alpha_t^*(\ell')/\partial T_t(\ell) > 0$ *for all* $\ell' \in \mathcal{N}(\ell)$*.*

Proof. When $T_t(\ell)$ falls, the spatial pool $N_t(R, \ell'; \theta)$ decreases for $\ell' \in \mathcal{N}(\ell, R_t^*)$. By Lemma 1(i), $R_t^*(\ell')$ increases; by Lemma 1(ii), $\sigma_g^{2*}(\ell')$ increases; by Lemma 3(i), $\alpha_t^*(\ell')$ decreases; by Lemma 5(iii), $\bar{p}_t(\ell')$ decreases; by Lemma 6, $\underline{z}_t^S(\ell')$ increases. All four changes push ℓ' toward the dead-market basin.

■

Proposition 3 (Global Games Uniqueness). *Suppose each broker observes* θ *with noise* $\tilde{\theta}_b = \theta + \varepsilon_b$*,* $\varepsilon_b \sim \mathcal{N}(0, \sigma_\varepsilon^2)$*. As* $\sigma_\varepsilon \to 0$*, the unique rationalizable equilibrium is a threshold strategy: broker* b *enters iff* $\tilde{\theta}_b \geq \hat{\theta}(\rho, \bar{p}, \ell)$*, where* V^B *from* (27) *satisfies the single-crossing property in* (θ, z_b) *and* $\hat{\theta}$ *is strictly decreasing in* ρ *and* $\bar{p}$*.*

5 Conclusion

We develop a dynamic theory of housing market collapse with three main results. Theorem 1 characterises the tipping-point structure: for any fixed population density ρ and disclosure level θ, the transaction dynamics admit a functioning-market attractor and a dead-market absorbing state, separated by a threshold $T^{\text{crit}}(\theta, \rho)$. As ρ declines, the functioning-market equilibrium eventually ceases to exist, and the collapse is irreversible. Higher disclosure shifts both thresholds favourably, expanding the set of ρ values consistent with a functioning market.

Theorem 2 establishes the dual-exit acceleration result: economic and demographic exit interact multiplicatively to compress the collapse timeline. Markets facing both thin-market conditions and aging broker populations collapse strictly faster than either channel alone predicts. Crucially, demographic exit alone—independent of profitability—can cause collapse when the existing broker cohort ages out and entry is insufficient.

Theorem 3 shows that disclosure is socially underprovided in the market equilibrium. The social benefit exceeds the private benefit through two externalities absent from broker optimisation: the information production externality (comparables benefit all future participants) and the market-survival externality (the planner values continued market existence). The marginal social benefit of disclosure is convex in θ, implying that completing a partially-implemented disclosure system yields disproportionately large returns.

Policy implications. The theory delivers three policy conclusions. First, the effective intervention window closes earlier than static models suggest: forward-looking brokers exit preemptively once the trajectory falls below the viability threshold, even before current profits turn negative. Second, monopoly pricing should be *tolerated* in thin markets (Proposition 1): forcing commissions below the monopoly level risks eliminating the last broker, converting a suboptimal market into a dead market. Third, disclosure generates positive spatial spillovers (Proposition 2): the social return to local reform exceeds the private return, providing theoretical justification for national disclosure mandates.

Future work. Three theoretical extensions are natural: (i) endogenous household migration (closing the demographic side), (ii) explicit network structure for spatial contagion (strengthening Proposition 2 to a theorem), and (iii) optimal mechanism design for disclosure (extending Theorem 3 to the full policy space).

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A Proofs of Additional Results

Remark on the Soft Intermediation Channel (Robustness to Direct Channel). See Remark 2 in Section 3.5.

Proof of Proposition 1 (Monopoly Exit Paradox). Under monopoly, broker profit is $\Pi_t^M = z_b(\tau_t^* \bar{p}_t - c^B(R_t^*))D_t - F(\ell)$. This is below $\bar{\Pi}$ iff $D_t < D_t^{\min}(\ell)$ as in (35). $D_t = s(\tau_t^*)b(\tau_t^*)\tilde{\alpha}_t \bar{S}_t \bar{B}_t / H \propto \rho^2 sb\tilde{\alpha}/H$. Setting $D_t = D_t^{\min}$ and solving for ρ gives $\underline{\rho}^M(\theta, \ell)$ as stated. $\partial \underline{\rho}^M / \partial \theta < 0$ because higher θ lowers $D_t^{\min}$ (via smaller R_t^* and c^B) and raises D_t (via higher $\tilde{\alpha}_t$, s_t, b_t), both reducing the minimum viable population.

Proof of Proposition 2 (Spatial Contagion). When $T_t(\ell)$ falls, the aggregate pool

$$N_t(R, \ell'; \theta) = \theta \pi R^2 \sum_{\ell''} T_t(\ell'')\rho(\ell'')$$

decreases for all ℓ' in the neighbourhood. By Lemma 1(i), $R_t^*(\ell')$ increases, which by Lemma 1(ii) raises $\sigma_g^{2*}(\ell')$, lowering $\tilde{\alpha}_t(\ell')$ and $h(T; \theta, \rho_{\ell'})$. The threshold $T^{\text{crit}}(\theta, \ell')$ shifts rightward (Theorem 1(ii)). Disclosure reform at ℓ raises $T_t(\ell)$, expanding N for neighbours; hence $\partial T^{\text{crit}}(\theta, \ell')/\partial T_t(\ell) < 0$.

Proof of Proposition 3 (Global Games Uniqueness). Standard application of the global games method of Carlsson and van Damme (1993) and Morris and Shin (2003). The broker entry game has a finite payoff $V^B(\theta)$ strictly increasing in θ (Lemma 1(ii) implies higher θ raises broker profit). Dominance regions exist: for $\theta < \underline{\theta}$, non-entry dominates; for $\theta > \bar{\theta}$, entry dominates. Iterated deletion of dominated strategies yields the unique threshold strategy. $\hat{\theta}(\rho, \bar{p})$ is decreasing in ρ and $\bar{p}$ because higher density and price raise V^B uniformly, shifting the entry threshold downward.

B Notation Table

Table 2: Complete Notation

Symbol	Definition	First defined
Primitives		
$\ell \in \mathcal{L}$	Location	Sec. 3.1
$\rho(\ell)$	Population density (fixed parameter)	Sec. 3.1
$H(\ell)$	Housing stock	Sec. 3.1

Continued on next page

(Table 2 continued)

Symbol	Definition	First defined
$\theta \in [0,1]$	Disclosure fraction	Sec. 3.1
$\bar{\tau}(\bar{p})$	Legal commission cap (upper bound)	Sec. 3.1
δ^{dir}	Direct-channel efficiency parameter	Assumption in Sec. 3.5
$F(\ell)$	Broker fixed operating cost	Sec. 3.7
κ^E	Broker entry cost	Sec. 3.7
$\bar{\Pi}$	Broker outside option	Sec. 3.7
State variables (core 2D system)		
$T_t(\ell)$	Transaction volume (deals/yr/1K units)	Sec. 3.8
$A_t^*(\ell)$	Active broker density (per 10K pop)	Sec. 3.8
State variables (supplementary)		
$\bar{p}_t(\ell)$	Mean transaction price	Sec. 3.6
$a_t(\ell)$	Akiya (vacancy) rate	Sec. 3.6
$Q_t(\ell)$	Neighbourhood quality index	Sec. 3.6
Information structure		
$N_t(R,\ell;\theta)$	Comparable sample size within radius R	Eq. (1)
$R_t^*(\ell;\theta)$	Agent-optimal search radius	Eq. (4)
σ_g^{2*}	Minimised estimation error	Eq. (2) at R_t^*
σ_β^2	Coefficient uncertainty	Eq. (2)
σ_h^2	Spatial hedonic drift rate	Eq. (2)
$n_U(\theta)$	Fraction of unobservable attributes	Eq. (2)
Matching and participation		
$\tilde{\alpha}_t$	Per-period matching probability	Eq. (14)
$\eta \in (0,1)$	Buyer bargaining weight (Nash)	Eq. (8)
$s_t(\ell;\theta)$	Seller participation rate	Eq. (19)
$b_t(\ell;\theta,\tau_t)$	Buyer participation rate	Eq. (21)
$\bar{S}_t, \bar{B}_t$	Potential seller/buyer pools	Eq. (16)
χ^S	Broker–seller contact rate	Eq. (19)
$\bar{r}$	Annual holding cost (seller)	Eq. (19)
φ	Listing cost	Eq. (19)
x_t	Market tightness ($A_t^*/(s_t\bar{S}_t/H)$)	Eq. (12)

Continued on next page

(Table 2 continued)

Symbol	Definition	First defined
$q(x_t)$	Broker per-seller matching probability	Eq. (12)
W_t^O	Seller expected wait time	Eq. (12)
Broker industry		
$z_b \sim F_z$	Broker idiosyncratic productivity	Sec. 3.7
ω_b	Principal age	Sec. 3.7
D_t	Annual deal count per broker	Eq. (24)
τ_t^*	Equilibrium commission $\leq \bar{\tau}$	Eq. (25)
$c^B(R) = c_0^B + c_R^B R^2$	Variable investigation cost	Eq. (24)
ϕ_t^S	State-dependent exit rate	Eq. (33)
ϕ_t^T	Time-dependent exit rate	Eq. (30)
ϕ_t^{tot}	Total exit rate	Eq. (33)
λ_t^E	Mass of potential entrants	Eq. (31)
$\underline{z}_t^S$	State-dependent exit threshold	Eq. (29)
Equilibrium objects		
$T^*(\theta, \rho)$	Functioning-market fixed point	Thm. 1
T^D	Dead-market fixed point	Def. 1
$T^{\text{crit}}(\theta, \rho)$	Separatrix (tipping point)	Thm. 1
$\rho^{\text{crit}}(\theta)$	Critical population density	Thm. 1
$\underline{\rho}^M(\theta, \ell)$	Min. population for monopoly viability	Prop. 1
$\hat{\theta}(\rho, \bar{p})$	Global Games collapse threshold	Prop. 3
Welfare		
$\widetilde{\mathcal{W}}_t(\ell; \theta)$	Relative social welfare	Def. 2
χ_{info}	Information externality value per transaction	Def. 2
e	Akiya externality cost (per unit)	Def. 2
θ^*	Socially optimal disclosure	Thm. 3
θ^{mkt}	Market-equilibrium disclosure	Thm. 3

C Numerical Illustration: Phase Diagram

To provide intuition for the quantitative magnitudes without introducing a calibration exercise, we plot the phase diagram of the reduced 2D system (T_t, A_t^*) under baseline parameter values consistent with the orders of magnitude in Japanese regional housing markets.

Parameter choices. We set $\theta \in \{0.30, 0.70, 0.95\}$, $\rho \in \{820, 400, 180\}$ (persons/km^2, corresponding to viable regional city, declining city, and rural municipality), $\bar{p} = 25$M¥, $F = 2.4$M¥, $\bar{\Pi} = 4.5$M¥, $\delta^{dir} = 0.06$.

Phase diagram. Figure 1 shows the nullclines $\{T_{t+1} = T_t\}$ and $\{A^*_{t+1} = A^*_t\}$ in (T, A^*) space for $\theta = 0.30$ and $\rho = 820$. The functioning-market equilibrium (T^*, A^{**}) is the upper stable intersection, the dead-market equilibrium $(T^D, 0) = (0, 0)$ is at the origin, and the separatrix $T^{\rm crit}$ is the unstable boundary.

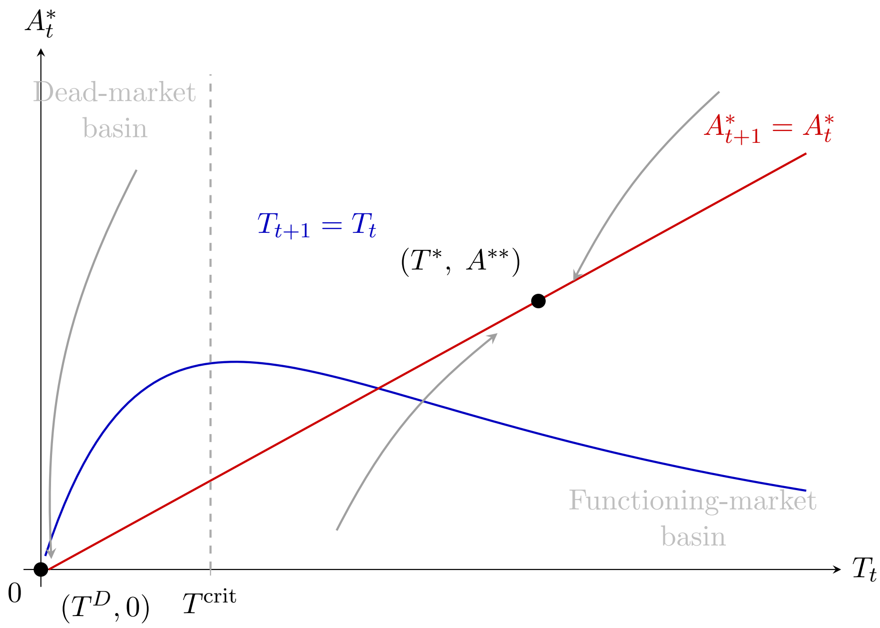

Figure 1: Phase diagram of the reduced 2D system $(T_t,\ A^*_t)$ for $\theta = 0.30$, $\rho = 820$ persons/km^2. **Blue curve**: transaction nullcline $(T_{t+1} = T_t)$, hump-shaped due to the information friction. **Red line**: broker density nullcline $(A^*_{t+1} = A^*_t)$, upward-sloping. **Filled circles**: stable fixed points—functioning market (T^*, A^{**}) and dead market $(T^D, 0) = (0, 0)$ at the origin. **Dashed vertical**: separatrix $T^{\rm crit}$ dividing the two basins of attraction. **Gray arrows**: representative trajectories converging to each attractor. *Key features*: (i) dead market is a corner attractor at $(T^D, 0) = (0, 0)$; (ii) raising θ from 0.30 to 0.70 shifts the blue nullcline rightward, moving the separatrix left; (iii) reducing ρ below $\rho^{\rm crit}(\theta)$ eliminates the functioning-market equilibrium; (iv) an aging shock $(\phi^T_t \uparrow)$ shifts the A^*-nullcline down, potentially crossing the separatrix. These correspond precisely to Theorems 1–2.