No. DP26-8

RCESR Discussion Paper Series

Measuring the Services of Durables and Owner Occupied Housing:
A Unified Framework and Forty Years of Tokyo Evidence

March 2026

W. Erwin Diewert,
University of British Columbia and UNSW Sydney
Chihiro Shimizu, Hitotsubashi University

RCESR

The Research Center for Economic and Social Risks
Institute of Economic Research
Hitotsubashi University

2-1 Naka, Kunitachi, Tokyo, 186-8603 JAPAN
http://risk.ier.hit-u.ac.jp/

Measuring the Services of Durables and Owner Occupied Housing:

A Unified Framework and Forty Years of Tokyo Evidence*

W. Erwin Diewert† and Chihiro Shimizu‡

March 2026

Abstract

Measuring owner-occupied housing services in the CPI is one of the most contested problems in official statistics. We nest five approaches—acquisitions, rental equivalence, user cost, financial user cost, and opportunity cost—within the identity $u_t = r_t^{\dagger} + c - \pi_t^{\dagger}$. Using 3.1 million Tokyo property records over 1986–2025, we show that conventional measures diverge sharply and can reverse sign across asset-price cycles. The opportunity cost approach eliminates negative user costs in all 480 sample months. The dominant CPI bias source is the *price* channel—procyclical acquisitions and sticky rents—not the weight channel. Our findings directly address the Eurostat HICP impasse.

JEL codes: C43, D12, E31, R21, R31

Keywords: Owner-occupied housing; CPI; cost-of-living index; financial user cost; rental equivalence; rent stickiness; hedonic regression; Tokyo; Eurostat HICP

---

*The authors thank seminar participants for helpful comments. Financial support from JSPS KAKENHI Grant (S) 24H00012 is gratefully acknowledged. All errors are the authors' own.

†School of Economics, University of British Columbia, Vancouver B.C., Canada, V6T 1Z1. E-mail: erwin.diewert@ubc.ca.

‡School of Social Data Science, Hitotsubashi University, 2-1 Naka, Kunitachi, Tokyo 186-8601, Japan. E-mail: c.shimizu@r.hit-u.ac.jp.

1 Introduction

Housing services are the largest component of household consumption in most advanced economies, yet their measurement in the Consumer Price Index (CPI) remains one of the most contested and consequential open problems in official statistics. Across the world's major metropolitan areas—Tokyo, London, New York, Sydney, Vancouver, and many others—residential property prices have risen sharply and persistently relative to household incomes, squeezing access to adequate shelter and generating broad social concern. Yet the macroeconomic policy response to this phenomenon is critically constrained by a measurement problem that has received far less attention than it deserves: the CPI, the primary instrument through which monetary authorities gauge inflation and calibrate policy, may itself be providing a systematically distorted signal of the true cost of housing services to households.

Accurate price measurement is not merely a statistical concern; it is a precondition for sound macroeconomic policy. Since the 1990s, most central banks have adopted explicit inflation-targeting frameworks in which the CPI serves as the operational target, with the policy rate adjusted to keep measured inflation within a narrow band—typically around 2 per cent. If the measured CPI is a biased proxy for the true cost of living, the policy rate will systematically deviate from its welfare-consistent level: the central bank will tighten too little when the true cost of living is rising, and too much when it is falling. As Goodhart (2001)[11] and subsequent literature have emphasised, housing-price booms tend to compress the CPI-measured shelter cost precisely because the investment motive dominates the consumption motive at the peak of the cycle—the opposite of what an inflation-targeting central bank needs to observe. A CPI that systematically understates shelter costs during asset-price booms will cause the central bank to hold interest rates lower than the true cost of living warrants, thereby reinforcing the boom, before abruptly reversing as the bust raises user costs. The distortion in the price signal thus becomes a source of policy *amplification* rather than stabilisation.

The difficulty of measuring housing services in the CPI arises from a structural feature of housing markets: owner-occupied dwellings are simultaneously consumption goods and investment assets. For tenant households, the monthly rent payment provides a direct and observable market price for one period's housing services. For the majority of households in most OECD economies who own their dwellings, no such transaction occurs: the household purchases a durable asset at a large up-front cost and consumes the implicit service flow over many years without making a period-by-period payment. The question of how to convert this asset purchase into a sequence of period service-flow prices has generated sustained methodological debate for over half a century—and has not been resolved.

Five approaches to this problem are now codified in the *Consumer Price Index Manual* (ILO et al., 2004[13]; IMF et al., 2025[19]): (i) the *acquisitions approach*, attributing the full asset price to the period of purchase; (ii) the *rental equivalence approach*, imputing a market rent; (iii) the *user cost approach*, computing the net financial cost of owning for one period; (iv) the *payments approach*, recording actual mortgage and tax outflows; and (v) the *opportunity cost approach*, taking the maximum of the user cost and the market rent. Despite the Manual's comprehensive treatment, no consensus has emerged on which approach is theoretically superior or empirically most reliable, partly because the approaches have seldom been compared systematically on the same data over a sufficiently long period. In Europe, this impasse has had direct institutional consequences: Eurostat has sought since 2014 to incorporate owner-occupied housing (OOH) into the Harmonised Index of Consumer Prices (HICP) but remains unable to reach methodological agreement, and the HICP continues to exclude OOH as of 2025. The experimental OOH-PI published by Eurostat uses the acquisitions approach—which, as this paper documents, is the approach most prone to procyclical mismeasurement.

This paper addresses that impasse directly. We make three contributions.

The **theoretical contribution** is to show that all five approaches can be expressed through a single identity $u_t = r_t^\dagger + c - \pi_t^\dagger$, where the pair $(r_t^\dagger, \pi_t^\dagger)$ characterises each approach's treatment of the opportunity cost of capital and expected capital gains. This unified representation clarifies the precise sense in which the approaches differ, establishes a theoretically motivated ordering under competitive equilibrium (Appendix B, Theorem B.1), and embeds the financial user cost (FUC) framework—which distinguishes between mortgage debt and home equity as distinct sources of financing—within the same structure.

The **empirical contribution** is threefold. First, we construct quality-adjusted hedonic price and rent indices for Tokyo condominiums and single-family houses at monthly frequency over the full 40-year period January 1986 to December 2025, using a rolling-window hedonic methodology applied to approximately 3.1 million property transactions and rental contracts from the Recruit/SUUMO platform—to our knowledge the largest integrated sale-and-rental micro-dataset assembled for any major city. Second, we implement all five measurement approaches on this common dataset, allowing a direct comparison free from the compositional and data-source differences that confound most existing comparative studies. Third, and most importantly, we quantify the divergence among the approaches across four sharply contrasting market regimes: the late-1980s Bubble, the Lost Decades Deflation of 1993–2003, the near-zero-rate period of 2014–2020, and the post-pandemic appreciation of 2021–2025. In the most extreme episode, the acquisitions approach registers annual price growth of 21.6 per cent while the Financial User Cost (Type B) simultaneously records −3.5 per cent—the two measures move in *opposite directions*, confirming that they measure fundamentally different objects.

The **policy contribution** addresses three interconnected problems that have blocked the adoption of welfare-consistent OOH measures in official statistics.

First, we resolve the negative user cost problem that has made user cost approaches appear impractical for CPI compilation. The Basic User Cost is negative in 36.2 per cent of the 480-month sample—174 months in total—including a sequence of 34 consecutive negative months during the Bubble (January 1987 to October 1989) with a minimum value of −73.2 per cent per annum. The Financial User Cost reduces but does not eliminate this problem. We demonstrate that the opportunity cost (OC) approach of Diewert (2008)[7]—$\text{OC}_t = \max\{\text{FUC}_t \cdot P_t^H,\ R_t^N\}$—eliminates negative values *entirely*: in the Tokyo data, all three OC variants record zero months with negative values across the full 480-month sample. This is the first large-scale empirical confirmation that the OC approach resolves the negative-value problem in practice.

Second, we document that the dominant source of CPI bias in the Tokyo data is the *price channel*—the procyclical acquisitions index and the lagging sticky-rent CPI—rather than the *weight channel* that has received most attention in the theoretical literature. The stickiness wedge between new-contract and incumbent rents averages −8.4 index points (2020 = 100) over the full sample, swinging from −17.4 during the Deflation to +4.0 in the Recent episode.

Third, we show that Tokyo, while extreme in the magnitude of its measurement problems, provides lessons directly applicable to other major cities. The structural condition that generates large measurement divergences—an expected appreciation rate that substantially exceeds or falls short of the financing cost—was present in the United States and United Kingdom during the 2000s, in Australia during the 2010s, and across most OECD metropolitan areas since 2021. We propose a concrete three-step implementation roadmap for national statistical offices, beginning with the immediately feasible transition from incumbent to new-contract rents and culminating in the publication of leverage-differentiated FUC indices that make visible the generational divide in housing costs.

The remainder of the paper is organised as follows. Section 2 reviews the relevant literature. Section 3 develops the unified theoretical framework. Section 4 constructs and compares the five measurement series on the Tokyo dataset. Section 5 analyses the FUC framework in detail, documents the negative user cost problem and its resolution via the OC approach, and derives welfare and policy implications. Section 6 concludes. Online Appendices A through F provide technical details on the computation of each approach (Appendix A), the theoretical relationships among approaches (Appendix B), depreciation models (Appendix C), hedonic regression specifications (Appendix D), the financial user cost (Appendix E), and the index construction methodology (Appendix F). Appendix G collects supplementary empirical results including HP-filter robustness, index-level comparisons, regime classification detail, and depreciation sensitivity analysis.

2 Literature Review

The measurement of owner-occupied housing (OOH) services has attracted sustained theoretical attention since Jorgenson (1963)[20] introduced the rental price of capital as the appropriate period cost of a durable asset. Building on this foundation, Hall and Jorgenson (1967)[34] formalised the user cost formula in the context of tax policy and investment, establishing the canonical expression $u_t = (r_t + \delta - \pi_t^e)P_t^H$ that underlies all subsequent work on durable-goods pricing. Diewert (1974)[29] extended the framework to consumer durables, connecting it to the cost-of-living index (COLI) of Könus (1924) and showing that the COLI-consistent period price for an owner-occupied dwelling is the opportunity cost of deploying wealth in the asset for one period, not the purchase price itself. Hulten (1990)[12] and Hulten and Wykoff (1981a, 1981b)[38][39] provided the empirical foundations for this approach, demonstrating that geometric depreciation fits housing data well and greatly simplifies aggregation across vintages (Appendix C reviews straight-line, geometric, and one-hoss-shay models, with empirical estimates from Tokyo data in Table C.7). The present paper builds directly on these foundations, adopting the COLI benchmark of Diewert (2005a)[6], the opportunity cost formulation of Diewert (2008)[7], and the heterogeneous-household extension of Diewert and Nakamura (2011)[9], while calibrating depreciation at $\delta = 0.020$ per year consistent with Diewert and Shimizu (2015, 2016)[30][31].

The five approaches to OOH measurement now codified in the *CPI Manual* (ILO et al., 2004[13]; IMF et al., 2025[19]) have distinct intellectual histories and serve different institutional purposes. The **acquisitions approach**, which attributes the full purchase price to the period of acquisition, is valued for its transparency but is theoretically inappropriate for long-lived assets, as it conflates the price of a claim on an infinite stream of shelter services with the price of one period's shelter (Appendix A, Section A.1). Goodhart (2001)[11] noted that CPIs relying on acquisitions-type measures fail to signal the true monetary conditions during housing booms, while Fenwick (2009, 2012)[4][5] and Eurostat (2017)[3] have advocated publishing multiple CPI variants to serve different analytical and policy purposes. The **rental equivalence approach**, endorsed by the System of National Accounts (Eurostat et al., 1993[2]) and adopted by the Bureau of Labor Statistics (1983)[27] in the United States, is theoretically sound when new-contract rents are used (Appendix A, Section A.2; Appendix F, Section F.6), but the practice of sampling incumbent rents introduces a systematic stickiness bias documented for the United States by Crone, Nakamura and Voith (2000, 2011)[1][28] and assessed across countries by Heston and Nakamura (2011)[35]. The **user cost approach**, implemented in official statistics by Iceland (Gudnason and Jonsdottir, 2011[33]), produces divergences from rents that Verbrugge (2008)[43] and Garner and Verbrugge (2011)[32] attribute to expected appreciation, tax advantages, and an illiquidity premium—components that are explicitly incorporated in the financial user cost (FUC) framework of Diewert, Nakamura and Nakamura (2009)[8] extended in Appendix E of this paper. Hill, Steurer and Waltl (2017)[44] show via simulation that the acquisitions and user cost approaches diverge most sharply from the rental equivalence benchmark at cyclical peaks, a prediction confirmed quantitatively in Section 4. The **payments approach**, which records actual mortgage outflows, was used in the UK Retail Prices Index until 1994 (Goodhart, 2001[11]) and remains in use in some countries (Appendix A, Section A.4.1); its principal deficiency—ignoring the equity opportunity cost and the capital-gain offset—is documented by Lebow and Rudd (2003)[40] and is embedded in the theoretical ordering of Appendix B (Theorem B.1).

A critical but under-resolved problem in the user cost literature is the possibility of **negative user cost values** during episodes of rapid house-price appreciation. If the expected capital gain π_t^e exceeds the sum of the financing rate and the carrying cost r_t+c, then $u_t = r_t+c-\pi_t^e < 0$, which is economically interpretable as a period in which the asset "pays" the owner to hold it, but is technically problematic as a CPI price relative. Verbrugge (2008)[43] documents this problem for the United States and argues for replacing realised appreciation with a smoothed long-run rate; Garner and Verbrugge (2011)[32] find that smoothing substantially reduces but does not eliminate negative values. Diewert (2008)[7] proposed the opportunity cost (OC) approach—$\mathrm{OC}_t = \max\{u_t \cdot P_t^H,\, R_t^N\}$—as a theoretically motivated resolution: since the rental flow R_t^N is always positive, the OC index is bounded below by zero by construction. Despite Diewert's (2008)[7] original proposal and its subsequent endorsement by Diewert and Nakamura (2011)[9], no study has yet tested whether the OC approach eliminates negative values in practice over an extended sample spanning both bubble and deflation episodes. The present paper fills this gap: using 480 months of Tokyo data, we show that the OC approach records zero months with negative values across the full sample, including during the most extreme phases of the late-1980s Bubble (Section 5). This empirical confirmation removes what has been the primary practical objection to user cost–based official statistics, and directly informs the implementation roadmap proposed in Section 6 (the leverage-type breakdown of negative UC incidence is documented in Appendix G, Section G.3).

The institutional context of the negative user cost problem is the **European HICP impasse**. Eurostat has sought since 2014 to incorporate OOH into the Harmonised Index of Consumer Prices but has been unable to reach methodological agreement among EU member states. The deadlock reflects two concerns: the absence of theoretical consensus on which approach is COLI-consistent, and the practical difficulty that user cost methods can produce negative values in boom episodes. As of 2025, the HICP continues to exclude OOH, and the experimental OOH-PI published by Eurostat uses the acquisitions approach. Eurostat (2017)[3] itself acknowledges the acquisitions approach as a compromise pending resolution of the methodological debate, and Eiglsperger (2021) documents the welfare costs of the continued exclusion. The present paper argues that both concerns can now be resolved—the first by the unified theoretical framework of Section 3, and the second by the empirical demonstration that the OC approach eliminates negative values in practice.

A second strand of literature directly relevant to this paper concerns **rent stickiness** and its implications for official price statistics. Calvo (1983)[21] introduced the random-opportunity model of price adjustment, in which the hazard of adjustment is constant and independent of the gap between the current price and its market-determined target. Shimizu, Nishimura and Watanabe (2010)[15] applied this framework to Japanese rental data and estimated a monthly adjustment hazard of $\hat{\Lambda}_0 = 0.025$—a Calvo parameter of 0.975—the highest degree of stickiness documented in any comparable study; by contrast, Hoffmann and Kurz (2002)[18] report 0.78 for Germany and Genesove (2003)[10] only 0.29 for the United States. Shimizu and Watanabe (2011)[16] show that this time-dependence of adjustment—as opposed to state-dependence— implies that accumulated deviations between market and contract rents do not self-correct rapidly, because the probability of a lease-event is orthogonal to the size of the gap. Suzuki, Asami and Shimizu (2021)[42] provide unit-level evidence that individual Tokyo units can carry rents 20–30 per cent below market levels for multi-year periods without triggering renegotiation. These findings establish the microeconomic foundations for the aggregate Calvo partial-adjustment equation estimated in Section 5 to characterise the stickiness wedge $\sigma_t = R_t^N - R_t^{\mathrm{CPI}}$ between the official CPI rent index and the COLI-consistent new-contract rent. A key implication, developed in Section 6, is that the transition from incumbent to new-contract rents in the rental equivalence approach is the single most cost-effective reform available to statistical agencies.

The quality-adjusted price and rent indices that underpin the empirical analysis of Section 4 are constructed using rolling-window hedonic regression following the methodology of Shimizu, Nishimura and Watanabe (2010a)[41], which is now the standard approach in the Japanese official residential property price index. The hedonic tradition for residential property traces from Bailey, Muth and Nourse (1963)[45] and is reviewed comprehensively by Hill (2013)[36]. The full hedonic specifications used in this paper are documented in Appendix D (land–structure decomposition) and Appendix F (rolling-window index construction). Diewert and Shimizu (2015)[30] applied the rolling-window method to Tokyo single-family house sales, simultaneously estimating depreciation rates and land price indexes; Diewert and Shimizu (2016)[31] extended the analysis to condominiums using the builder's model that decomposes total property value into land and structure components (Appendix D, equations (D.2)–(D.3)). Hill, Scholz, Shimizu and Steurer (2018)[37] confirm that rolling-window approaches outperform fixed-base and repeat-sales methods in thin markets, while the simultaneous availability of sale and rental listings from the Recruit platform—exploited here and in Shimizu, Diewert, Nishimura and Watanabe (2012)[14]—eliminates the compositional inconsistencies that confound most comparative studies.

The price-to-rent ratio P_t^H/R_t, which serves as the primary empirical indicator of the divergence between the welfare and investment layers of housing in this paper, has been the focus of an extensive literature since Poterba (1984)[22] modelled it as the reciprocal of the user cost rate in a no-arbitrage equilibrium. Campbell, Davis, Gallin and Martin (2009)[23] document a secular rise in the US price-to-rent ratio from the mid-1990s to 2006 that far exceeds what changes in fundamentals can explain, while Himmelberg, Mayer and Sinai (2005)[24] show that user cost adjustments for interest rates and expected appreciation can account for much of the apparent overvaluation. For Japan, Shimizu and Nishimura (2007)[25] decompose the 1986–2000 Tokyo price cycle into fundamental and speculative components, finding that the late-1980s bubble was driven by self-fulfilling appreciation expectations with no counterpart in housing demand fundamentals.

Finally, the policy stakes of OOH measurement are high under inflation-targeting frameworks, where a systematic bias in the shelter component of the CPI translates directly into miscalibrated policy rates. Goodhart (2001)[11] was among the first to document this channel, and Lebow and Rudd (2003)[40] estimate that the US CPI OOH bias contributed approximately 0.25 percentage points per year of downward mismeasurement, sufficient to shift the policy rate by 25–50 basis points under a standard Taylor rule. For Japan, the combination of extreme rent stickiness, a home-ownership rate of approximately 61 per cent, and the largest peacetime housing bubble of the twentieth century (Shimizu and Nishimura, 2007[25]) creates conditions under which the gap between the official CPI shelter measure and a COLI-consistent alternative is both large and cyclically systematic. The Bank of Japan's prolonged zero-interest-rate policy of the 1990s and 2000s has been interpreted in part as a consequence of a deflationary CPI signal that failed to capture the true cost of shelter; see Hoshi and Kashyap (2004)[26]. Sections 4 and 5 quantify these divergences over the full 40-year Tokyo sample and Section 6 draws the implications for monetary policy, statistical reform, and the European HICP impasse.

3 Measuring Housing Services: Theory and Approaches

3.1 The Theoretical Benchmark and the Three Approaches

The Könus Price Index and Its Welfare Foundation

The theoretical foundation of any Consumer Price Index is the *cost-of-living index* (COLI) of Konüs (1924). Let the household's preferences be represented by a utility function $U(\mathbf{q}^{\text{nd}}, h)$ defined over a vector of non-durable goods and services $\mathbf{q}^{\text{nd}}$ and a scalar h representing one period's worth of constant-quality housing services. The associated expenditure function is $e(\mathbf{p}^{\text{nd}}, p^h, u) \equiv \min\{(\mathbf{p}^{\text{nd}} \cdot \mathbf{q}^{\text{nd}}) + p^h h : U(\mathbf{q}^{\text{nd}}, h) \geq u\}$, where p^h is the period price of one unit of housing services. The Könus COLI between reference period 0 and comparison period t, evaluated at the reference utility level u^*, is

$$P^K(\mathbf{p}^0, \mathbf{p}^t, u^*) \equiv \frac{e(\mathbf{p}^{t,\mathrm{nd}},\ p^{h,t},\ u^*)}{e(\mathbf{p}^{0,\mathrm{nd}},\ p^{h,0},\ u^*)} \tag{3.1}$$

The COLI has a precise welfare interpretation: it answers the question "by what factor must the household's budget increase to remain exactly as well off in period t as in period 0?" Diewert (1976)[6] showed that a *superlative* index formula—one that is exact for a flexible functional form of the expenditure function, such as the Fisher ideal or Törnqvist index— provides a second-order approximation to the COLI using only observed prices and expenditure shares, without requiring knowledge of the household's preferences.

The central requirement of (3.1) is that $p^{h,t}$ be a well-defined *scalar period price* for one unit of constant-quality housing services consumed in period t. For non-durable goods and services, the period of payment coincides with the period of consumption, so the market transaction price directly satisfies this requirement. For consumer durables—including housing—payment and consumption are decoupled, and $p^{h,t}$ must be derived from the structure of the durable's market rather than read off directly from a single market transaction. This is the fundamental measurement problem that motivates the five approaches discussed in this paper.

Durables with Short Service Lives: The Acquisitions Approach and the Rental Market

For consumer durables with a *short service life*, the decoupling of payment and consumption is quantitatively minor, and two approaches provide adequate approximations to the COLI-consistent price.

The first is the *acquisitions approach*: the full purchase price is attributed to the period of acquisition. The theoretical justification rests on the observation that for a durable with a short service life L, the present value of all future service flows is approximately proportional to the current purchase price. More precisely, from the one-hoss-shay depreciation model (Online Appendix C, equation (C.17)), the per-period user cost satisfies

$$u^0 = \frac{(1-\gamma)\,P_0^0}{1-\gamma^L}, \qquad \gamma \equiv \frac{1+i^0}{1+r^0} \tag{3.2}$$

As $L \to 1$, $u^0 \to P_0^0$: the user cost collapses to the purchase price, and the acquisitions approach becomes exact. For small L (say, clothing with $L = 2$–3 years), the error is modest. Online Appendix B shows that for a durable with $\delta = 0.20$, the ratio of the user cost value to the acquisitions value is approximately 1.11 (Online Appendix B, equation (B.14)): the two measures differ by only 11 per cent, well within the margin of practical measurement error. The acquisitions approach is therefore a defensible simplification for short-lived durables, and it is universally adopted by statistical agencies for goods such as clothing, appliances, and furniture.

The second approach applies when a well-organised *rental or leasing market* for the durable exists. In that case, the observed rental price R_t for one period's use of the durable directly measures the current market value of one unit of service, and is the COLI-consistent period price without further imputation. The theoretical justification follows from the landlord's zero-profit condition. A risk-neutral lessor who purchases the durable at price P_t, leases it for one period at rate R_t, and recovers the depreciated resale value $(1-\delta)P_{t+1}$ earns zero profit in competitive equilibrium if and only if $R_t = P_t(1+r_t) - (1-\delta)P_{t+1}$, which is precisely the user cost formula (Online Appendix B, equation (B.4)). The rental price therefore equals the user cost in equilibrium, providing a direct and observable measure of the COLI-consistent period price. For automobiles in countries with developed leasing markets, this approach is often preferred; see ILO et al. (2004)[13].

The Special Character of Housing: The Dual Nature of Consumption and Investment

Owner-occupied housing is distinguished from other consumer durables by two fundamental characteristics that jointly invalidate both the simple acquisitions approach and the uncorrected rental equivalence approach as COLI measures.

The first is *extreme longevity*: the service life of a dwelling typically exceeds fifty years. For such an asset, the gap between the purchase price—which equals the present discounted value of all future service flows—and the current period's service-flow cost is large and highly sensitive to the discount rate and to expectations about future prices. The formula analogous to (3.2) gives a ratio of user cost value to acquisitions value of approximately 1.68 for housing under plausible calibrations ($\delta = 0.02$, $r^* = 0.03$, $g = 0.01$; Online Appendix B, equation (B.15)), versus 1.11 for short-lived durables. The acquisitions approach therefore understates the true cost of housing consumption by 40 per cent or more.

The second, and more fundamental, distinction is the *dual nature of housing as simultaneously a consumption good and an investment asset.* A household purchases a dwelling to obtain shelter services— warmth, space, security, and locational access—but simultaneously acquires a financial claim whose value fluctuates with market conditions. These two motivations coexist in every housing purchase, and their relative importance varies systematically over the housing cycle in a way that has direct consequences for the COLI.

Formally, let P_t^H denote the quality-adjusted price of a dwelling, δ the geometric structure depreciation rate, r_t the household's nominal opportunity cost of capital, τ the property tax rate, m the maintenance rate, and $\pi_t^e \equiv (P_{t+1}^{H,e} - P_t^H)/P_t^H$ the expected one-period capital gain rate. The COLI-consistent period price of housing services—the *user cost rate*—is

$$u_t \;\equiv\; r_t \;+\; \underbrace{(\delta + \tau + m)}_{c} \;-\; \pi_t^e \tag{3.3}$$

and the corresponding monetary user cost is $U_t = u_t \cdot P_t^H$. The three terms in (3.3) capture, respectively, the forgone financial return on the equity tied up in the dwelling, the net physical cost of occupancy, and the offsetting investment gain from holding an appreciating asset.

The dual nature of housing enters through the term π_t^e. When house prices are expected to *fall*—or in a period of actual price decline—households that purchase do so primarily for the consumption value of shelter: the investment return is negative or zero, so only those who place a sufficiently high utility value on owner-occupancy relative to renting will choose to buy. In this regime, the consumption motive dominates and the user cost u_t is high.

When house prices are expected to *rise* substantially, investment-motivated buyers enter the market in pursuit of capital gains: the expected appreciation π_t^e offsets the financial carrying cost r_t, reducing the user cost. In the extreme case of a speculative bubble, π_t^e may exceed $r_t + c$, driving the user cost negative—meaning that the dwelling literally pays the owner to hold it. The equilibrium condition that in a frictionless competitive market rent equals the user cost (Online Appendix B, equation (B.4)) then implies that new-contract rents are also driven down by anticipated capital appreciation. This produces a situation in which the asset price and the rental price move in *opposite directions*: the asset price rises as buyers bid up values, while the rental price falls as the investment subsidy reduces the cost of providing rental services.

The mathematical relationship between the asset price and the service-flow cost follows from the present-value identity (Online Appendix B, equation (B.16)):

$$P_t^H \;=\; \sum_{s=0}^{\infty} \frac{(1-\delta)^s\, u_{t+s}\, P_{t+s}^H}{(1+r)^s} \tag{3.4}$$

The current asset price equals the present discounted value of *all future* user costs, not just the current one. In steady state with constant expected appreciation rate π^e, (3.4) simplifies to $u_t = r - \pi^e + \delta$ (Online Appendix B, equation (B.17)), confirming that the service-flow cost and the asset price are related but fundamentally distinct objects: the asset price summarises the discounted future, while the COLI requires only the current period's service cost.

Why the Acquisitions Approach is Inadequate for Housing

The dual nature of housing and its extreme longevity together imply that the acquisitions approach is unsuitable for measuring housing services in the COLI, for both theoretical and practical reasons.

Theoretical problem. The acquisitions approach treats the purchase of a dwelling as if it were the purchase of a non-durable: the full asset price P_t^H is attributed to the period of acquisition. This conflates the price of a claim on an infinitely-lived stream of shelter services with the price of one period's shelter. Under the present-value identity (3.4), the asset price moves with *expectations about all future user costs and capital gains.* A rise in expected future capital gains—π^e increasing— raises P_t^H via (3.4) while simultaneously *reducing* u_t via (3.3). The acquisitions price and the COLI-consistent price therefore move in *opposite directions* during episodes of rapid appreciation: the acquisitions index registers inflation at the exact moment when the true cost of owner-occupancy is falling.

Expenditure weight problem. A second theoretical problem concerns the expenditure weight. The acquisitions approach assigns the housing component a weight equal to the value of net new purchases of dwellings, whereas the COLI-consistent approach assigns a weight equal to the total value of housing services consumed from the entire existing stock. Online Appendix B (equation (B.14)) derives the exact ratio of the two weights:

$$\frac{V_U^0}{V_A^0} = \frac{(1+g)(r^*+\delta)}{g+\delta} \tag{3.5}$$

Table 1 calibrates this ratio for housing and for two representative short-lived durables under plausible parameter values.

Table 1: Ratio of User-Cost Value to Acquisitions Value: V_U^0/V_A^0 under Alternative Calibrations

Asset type	δ	r^*	g	V_U/V_A	CPI weight gap
Housing (baseline)	0.020	0.030	0.010	**1.68**	+68%
Housing (high r^*)	0.020	0.050	0.010	**2.35**	+135%
Housing (low δ)	0.010	0.030	0.010	**2.02**	+102%
Appliances	0.100	0.030	0.010	1.22	+22%
Clothing	0.200	0.030	0.010	1.11	+11%

Table 1 shows that for housing under baseline parameters, the acquisitions approach assigns a weight that is only 60 per cent of the COLI-consistent weight. As a result, the acquisitions approach not only mismeasures the *price* of housing services; it also dramatically *understates the importance of housing in the overall CPI.* For appliances and clothing, the gap is small enough to be of limited practical concern—a finding that justifies the universal application of the acquisitions approach to non-housing durables.

Measurement problem: procyclical volatility. Even setting aside the theoretical misspecification, the acquisitions approach introduces severe measurement distortions because it mixes the consumption and investment motives in a time-varying way. To illustrate the magnitude, consider two extreme episodes drawn from the Tokyo data analysed in this paper:

- **Bubble peak (1989–1990).** Expected annual capital gains $\pi_t^e \approx +0.15$ (15 per cent per year). The user cost rate becomes $u_t = 0.030 + 0.034 - 0.150 = -0.086$: the dwelling pays the owner approximately 8.6 per cent of its value per year to hold it, and the COLI-consistent period price of shelter is effectively *negative.* Yet the acquisitions price index rose by about 15 per cent, registering severe inflation at the exact moment when the true cost of owner-occupancy was at its historical minimum.

- **Post-bubble deflation (1993–2003).** Expected capital losses $\pi_t^e \approx -0.05$ per year.

The user cost rate rises to approximately $u_t \approx 0.114$, i.e., the true cost of owner-occupancy is elevated to nearly double its long-run average. Yet the acquisitions price index fell sharply, signalling deflation at the exact moment when the true shelter cost was highest.

These two episodes exemplify a general principle: a CPI based on the acquisitions approach *systematically overstates* the true cost of shelter during asset-price booms and *understates* it during busts—precisely inverting the welfare signal that monetary policy requires. This inversion is not a matter of random measurement error; it is the theoretically predicted consequence of conflating an asset price with a service-flow cost in a market where the investment and consumption motives for holding housing are simultaneously operative.

The theoretical and measurement problems together imply that the acquisitions approach should not be used for owner-occupied housing in a COLI-consistent price index. The appropriate measure is either the user cost U_t, the adjusted new-contract rent $R_t^{\text{new,adj}}$, or—if one wishes to bound the true cost from above—the unadjusted new-contract rent R_t^{new}. The formal ordering of all five approaches under the COLI criterion is derived in Online Appendix B (Theorem B.1) and is taken up empirically in Section 4.

3.2 The Rental Equivalence Approach

Rent as the Welfare-Layer Price

From the perspective of the Könus COLI, the ideal period price of housing services is the minimum expenditure per period that allows a household to obtain one unit of constant-quality shelter at current market conditions. In the taxonomy of Section 3.1, this is the service-flow price $p^{h,t}$, not the asset price P_t^H. For a household that rents its dwelling, the monthly contract rent R_{it} is a direct cash payment for exactly one period's worth of shelter services, and—when rents are competitively determined and quality-adjusted—it provides the most natural observable proxy for $p^{h,t}$. In this sense, the rental equivalence approach operates at the *welfare layer* of housing consumption: it measures what the market charges for shelter as a flow service, abstracted from the investment value of the underlying asset. This is its principal theoretical virtue, and it is why the rental equivalence approach is endorsed by the System of National Accounts [2] and adopted in the official CPIs of Japan, the United States, and most OECD countries.

As established in Section 3.1, however, the rental price and the user cost diverge systematically during asset-price cycles precisely because the housing market operates at two layers simultaneously: the welfare layer (shelter services) and the investment layer (capital gains). When expected capital appreciation is high, the investment subsidy π_t^e reduces the user cost below the competitive rent, attracting investment buyers and compressing new-contract rents; when prices are expected to fall, the absence of the capital-gain offset raises the user cost above rents. The rental equivalence approach is therefore COLI-consistent *only if* new-contract rents are used and only to the extent that frictions in the rental market are controlled for. In practice, large frictions exist—both in the structure of official rent statistics and in the micro-dynamics of individual lease contracts—that cause measured rents to diverge substantially and persistently from the COLI benchmark. This section documents those frictions.

Institutional Frictions: Public Housing and Statistical Measurement

A first class of frictions arises from institutional features of rental markets that cause the *distribution of rents observed in official statistics* to differ from the competitive market price of shelter.

In Japan and many European countries, a significant share of the rental stock consists of *public or quasi-public housing*— dwellings provided by central and local governments, or by employers, at below-market rents for equity or social-policy reasons. In Japan, *kōeki jutaku* (public housing) and *kōsha jutaku* (public corporation housing) accounted for approximately 7 per cent of the total dwelling stock as of the 2018 Housing and Land Survey. Rents in this segment are administratively determined and adjusted infrequently, often falling far below prevailing market levels during periods of house-price appreciation. If official CPI rent surveys include public housing units in the sample without appropriate adjustment, the measured rent index will be downward-biased relative to the market opportunity cost of shelter—precisely the COLI-consistent price that monetary policy requires.

A second institutional friction arises from the *design of rent surveys* themselves. Most national statistical agencies—including Japan's Statistics Bureau (which compiles the CPI) and the U.S. Bureau of Labor Statistics—survey *incumbent tenants*, asking what rent the household is currently paying. This practice captures the *stock* of existing rental contracts rather than the *flow* of newly concluded transactions. Because most existing contracts were signed months or years ago at prices that reflected then-prevailing market conditions, a survey of incumbent rents will, by construction, reflect the history of past market conditions rather than current ones. The CPI for rent thus embeds an inherent backward-looking bias that is particularly severe during episodes of rapid price change.

Micro-Dynamics of Rent Adjustment: The Stickiness Mechanism

The deeper source of the divergence between measured rents and the COLI price is the *stickiness of individual rental contracts*. To understand this mechanism formally, define two indicator variables following Shimizu et al. (2010)[15]: let I_{it}^N equal one if unit i experiences a tenant turnover and a new contract is signed in period t (zero otherwise), and let I_{it}^R equal one if an existing contract for unit i is renewed in period t (zero otherwise). The probability that the rent for unit i is *unchanged* in period t can then be decomposed into three terms:

$$\begin{aligned}\Pr(\Delta R_{it}=0) \;=\; & \big[1-\Pr(I_{it}^{N}=1)-\Pr(I_{it}^{R}=1)\big] \\ & +\;\Pr(\Delta R_{it}=0 \mid I_{it}^{N}=1)\cdot\Pr(I_{it}^{N}=1) \\ & +\;\Pr(\Delta R_{it}=0 \mid I_{it}^{R}=1)\cdot\Pr(I_{it}^{R}=1) \qquad (3.6)\end{aligned}$$

Equation (3.6) decomposes the probability into three components: term (i) covers periods with no contract event; term (ii) covers new-signed contracts that nonetheless leave the rent unchanged; term (iii) covers rollover renewals with no rent adjustment. The *necessary condition* for any rent change is that either a tenant turnover producing a new-signed contract ($I_{it}^{N}=1$) or a lease renewal producing a rollover contract ($I_{it}^{R}=1$) must occur. In the absence of both events—term (i) in (3.6)— the rent is frozen at its contract value regardless of how far market conditions have moved.

New-signed contracts. When a tenant vacates and a new tenant is sought, the landlord is free to set the rent at whatever level the current market will bear. The new-contract rent R_{it}^{N} therefore reflects the *current market opportunity cost of shelter*—the marginal price of a freshly negotiated lease in prevailing market conditions. It is this price that satisfies the COLI requirement: in a competitive market, a household choosing between owning and renting at the margin will face R_{it}^{N} as the relevant alternative, so R_{it}^{N} is the COLI-consistent price $p^{h,t}$ of equation (B.2).

Rollover contracts. When an existing tenant renews their lease—the *rollover contract*—the outcome is very different. Under Japan's Land Lease and House Lease Law (*Shakuya Hō*), a landlord cannot unilaterally raise the rent of an existing tenant except under narrow and difficult-to-invoke conditions. Shimizu, Nishimura and Watanabe (2010)[15] find, using 15,639 contract documents from a major Tokyo property management company (Daiwa

Living), that the probability of *no rent change* at rollover is $\Pr(\Delta R_{it} = 0 \mid I_{it}^R = 1) = 0.970$: fully 97 per cent of rollover renewals in March 2008 produced an unchanged rent. Moreover, when rent *does* change at rollover, the adjustment is strongly asymmetric: it is relatively easy for the landlord and tenant to agree to *lower* the rent when the existing rent exceeds the market level, but it is extremely difficult for the landlord to propose a rent *increase* to an existing tenant even when the market has moved sharply upward—because any such proposal is perceived as a violation of the implicit long-term relationship.

Aggregate probability of no rent change. Combining the three terms of (3.6) using empirical estimates from Shimizu et al. (2010)[15]— monthly turnover probability $\Pr(I_{it}^N = 1) = 0.034$, monthly rollover probability $\Pr(I_{it}^R = 1) = 0.038$, probability of no change given new contract $\Pr(\Delta R_{it} = 0 \mid I_{it}^N = 1) = 0.755$, and $\Pr(\Delta R_{it} = 0 \mid I_{it}^R = 1) = 0.970$—yields an aggregate monthly probability of no rent change of approximately 0.991, or equivalently, an *annual* probability of no rent change of **0.893**. As Table 2 shows, this degree of stickiness is exceptional by international standards.

Table 2: Annual Probability of No Rent Adjustment: International Comparison

Country	$\Pr(\Delta R_{it} = 0)$ per year	Source
Japan	0.893	Shimizu, Nishimura and Watanabe (2010)[15]
Germany	0.780	Hoffmann and Kurz (2002)[18]
United States	0.290	Genesove (2003)[10]

Rent stickiness in Japan is three times as high as in the United States and substantially higher than in Germany—a pattern reflecting both the distinctive legal protection of incumbents under Japanese tenancy law and the prevalence of long-term implicit contracts between landlords and tenants.

Time-Dependent Rather Than State-Dependent Adjustment

The stickiness documented above would be less consequential for the COLI if rent adjustments, when they did occur, were triggered by deviations of the actual rent from its market-determined target level. That is, if adjustment were *state dependent*—governed by the gap $x_{it} \equiv \log(R_{it}/R^*_{it})$ between the actual rent and the hedonic-imputed target rent R^*_{it}— large accumulated deviations would eventually trigger large corrections, and the aggregate rent index would track market conditions reasonably well even if individual adjustments were infrequent.

Shimizu et al. (2010)[15] test this hypothesis directly by estimating the adjustment hazard function

$$\begin{aligned} \Lambda(x) \;\equiv\; & \Pr(\Delta R_{it} \neq 0 \mid I^N_{it} = 1,\, X_{it} = x)\, \Pr(I^N_{it} = 1 \mid X_{it} = x) \\ & +\; \Pr(\Delta R_{it} \neq 0 \mid I^R_{it} = 1,\, X_{it} = x)\, \Pr(I^R_{it} = 1 \mid X_{it} = x) \end{aligned} \tag{3.7}$$

using the 718,811 listing records of the Recruit dataset (Tokyo 23 wards, 1986–2006). The key finding is that while the conditional probabilities of rent adjustment given a contract event are weakly state-dependent, the *occurrence of the contract event itself* is not:

- The monthly probability of tenant turnover, $\Pr(I^N_{it} = 1 \mid X_{it} = x) \approx 0.010$, is *independent of x*. Turnovers are caused by exogenous life events—job transfer, marriage, childbirth, retirement—that are orthogonal to the gap between actual and market rent. This is classic *time-dependent* behaviour in the sense of Calvo (1983): the landlord receives a random opportunity to reset the price, but this opportunity does not arrive more frequently when the existing price is further from its target.

- The monthly probability of lease renewal, $\Pr(I^R_{it} = 1 \mid X_{it} = x) = 1/24 \approx 0.042$, is by legal and contractual convention fixed at a two-year cycle, entirely independent of market conditions.

Because neither the probability of turnover nor the probability of renewal depends on x, the aggregate adjustment hazard $\Lambda(x)$ is nearly flat in x, as confirmed by the estimated values reported in Shimizu et al. (2010): $\Lambda(x) \approx 0.008$ for $x \in (-0.4, 0.2]$ and 0.011 for $x \in (0.2, 0.4]$—essentially constant.

The aggregate implication is immediate. When $\Lambda(x) = \Lambda_0$ (time-dependent, constant hazard), the aggregate rent dynamics follow the Calvo (1983) partial adjustment equation:

$$R_t \;=\; \Lambda_0 \, R_t^* \;+\; (1 - \Lambda_0) \, R_{t-1} \tag{3.8}$$

where R_t is the aggregate rent index (proxied by the CPI for rent), R_t^* is the target (market) rent index estimated from new-contract hedonic regressions, and $1 - \Lambda_0$ is the Calvo parameter—the probability that a given unit does *not* receive a random rent-reset signal in the period. Shimizu et al. (2010)[15] estimate $\hat{\Lambda}_0^{\text{micro}} = 0.025$ at the quarterly frequency from micro data, implying a **Calvo parameter of 0.975**—an extraordinarily high degree of price rigidity by any macroeconomic standard. The macro-level OLS estimate of $\hat{\Lambda}_0^{\text{macro}}$ from the aggregate time-series of incumbent versus new-contract rents in our Tokyo dataset is 0.0011 (s.e. 0.0009), implying a Calvo parameter of 0.9989—even more extreme than the micro estimate, reflecting the near-zero probability that any given incumbent contract is renegotiated in a given month at the aggregate level. These two estimates ($\hat{\Lambda}_0^{\text{micro}} = 0.025$ at the quarterly frequency and $\hat{\Lambda}_0^{\text{macro}} = 0.0011$ at the monthly frequency) are conceptually distinct: the micro estimate captures the hazard for a given *individual* contract, while the macro estimate captures the aggregate adjustment speed of the *stock* of incumbent rents, which is further attenuated by the multi-year tenure of existing leases.

The consequence for the CPI is quantitatively large. Shimizu et al. (2010)[15] simulate counterfactual CPI inflation under the assumption that Japanese rent flexibility were equal to that of the United States ($\Lambda_0^{US} \approx 0.25$, compared with the Japanese $\Lambda_0^{JP} \approx 0.025$). They find that CPI inflation during the bubble period (late 1980s) would have been *higher by* *approximately 1 percentage point per year*, while inflation during the post-bubble deflation (1990s) would have been *lower by more than 1 percentage point per year*— with deflation commencing approximately one year earlier than the official CPI recorded. These magnitudes are economically significant for monetary policy: a 1–2 percentage point gap between the measured and true inflation rate is large enough to shift the policy rate by 50–100 basis points under a standard Taylor rule.

Why the CPI Manual Recommends New-Signed Contract Rents

The analysis above provides a rigorous theoretical basis for the recommendation of the CPI Manual (IMF et al., 2025[19]; ILO et al., 2004[13]) that the rental equivalence approach should be implemented using *new-signed contract rents* rather than incumbent (roll-over) rents.

The argument proceeds in three steps.

Step 1: The COLI requires the marginal cost of shelter. From the Könus COLI (3.1), the housing price entering the expenditure function must be the *minimum market cost* of obtaining one unit of constant-quality shelter in period t. This is the opportunity cost that a utility-maximising household faces when deciding whether to rent or to own in period t. For a household making a housing decision *at the margin in period* t, the relevant price is the rent that would be negotiated in a *freshly signed lease* as of period t— not the average of all outstanding contracts, many of which were priced in previous periods under different market conditions. Formally, the COLI-consistent rental price is the new-contract rent R_t^N that satisfies the landlord's zero-profit condition in period t:

$$R_t^N \;=\; \left[r_t - \pi_t^e + \delta(1+\pi_t^e) + \tau + m_t + \nu_t\right] P_t^H \tag{3.9}$$

(Online Appendix B, equation (B.5)). The incumbent rent R_t^{contract}, by contrast, reflects the conditions prevailing when that contract was signed—say, k periods ago—and is therefore $R_t^{\text{contract}} \approx R_{t-k}^N$, an outdated price that does not satisfy (3.9).

Step 2: The stickiness wedge as a measurement bias. Define the stickiness wedge as $\sigma_t \equiv R_t^N - R_t^{\text{contract}}$ (Online Appendix B, equation (B.8)). From the Calvo dynamics (3.8), the aggregate gap between the new-contract rent index and the incumbent-rent index is

$$R_t^N - R_t \;=\; R_t^* - R_t \;=\; \frac{1-\Lambda_0}{\Lambda_0}\big(R_t^* - R_{t-1}\big) \;\approx\; \frac{1-\Lambda_0}{\Lambda_0} \cdot \dot{R}_t^* \tag{3.10}$$

where $\dot{R}_t^*$ is the rate of change of the target rent. With $\Lambda_0 = 0.025$ (Japan), the multiplier $(1-\Lambda_0)/\Lambda_0 \approx 39$: a 1 per cent monthly increase in the target rent produces a lag of 39 months before the aggregate incumbent-rent index fully reflects the change. With $\Lambda_0 = 0.25$ (US), the multiplier is only 3, implying a lag of 3 months. The CPI for rent in Japan therefore understates the COLI-consistent price during appreciation episodes by an amount that accumulates over years, not months.

Step 3: Only new-contract rents are COLI-consistent. Combining Steps 1 and 2: the incumbent-rent CPI satisfies $R_t^{\text{CPI}} = R_t \neq R_t^N = p_{\text{COLI}}^{h,t}$ whenever $\sigma_t \neq 0$, i.e., whenever rents are sticky and the market is not in a steady state. The bias is *positive* (CPI understates COLI) during appreciation episodes when $\sigma_t > 0$, and *negative* (CPI overstates COLI) when $\sigma_t < 0$ during price declines. New-signed contract rents R_t^N, by contrast, reflect current market conditions by construction: they are the equilibrium prices of freshly negotiated leases and therefore satisfy the COLI requirement (3.9) without further adjustment. *It is for this reason that the CPI Manual (IMF et al., 2025[19]) recommends that statistical agencies implement the rental equivalence approach using new-contract rents*, and why this paper uses the new-contract rent index constructed from the Recruit dataset as the empirical counterpart of R_t^N throughout the empirical analysis. The empirical contrast between new-contract and incumbent rents in the Tokyo data is documented in Appendix F, Section F.6.

3.3 The Financial User Cost

From Basic User Cost to Financial User Cost

The Basic User Cost introduced in Section 3.1 prices the cost of owner-occupancy using a single interest rate as the opportunity cost of capital. This simplification, while analytically tractable, obscures an important feature of housing finance: the purchase of a dwelling is funded through a combination of the household's own equity and borrowed funds, and these two sources of capital carry *fundamentally different costs.*

Housing markets are deeply intertwined with credit markets. When financial institutions expand mortgage lending—lowering borrowing rates, relaxing collateral requirements, or increasing loan-to-value ratios—the effective cost of financing a dwelling falls, stimulating demand and driving up asset prices. Conversely, when credit conditions tighten, the cost of financing rises and prices fall. This credit-housing nexus implies that the opportunity cost of capital in the user cost formula is not a single exogenous rate but a *financing-structure-dependent composite* that moves with both monetary policy and credit market conditions. The *Financial User Cost* (FUC) framework, developed by Diewert, Nakamura and Nakamura[8] and extended in Online Appendix E, makes this structure explicit.

Two Qualitatively Distinct Rates of Return

The key conceptual contribution of the FUC framework is the recognition that the two components of housing finance carry rates of return with *qualitatively different economic properties.*

Mortgage rate r_t^m. The mortgage rate is the contractual interest rate paid by the household to a financial institution for borrowed funds. It is determined in the credit market and reflects the lender's cost of funds, credit risk, and intermediation spread. Crucially, r_t^m is a *debt rate*: it is fixed (or pre-specified) at the time of borrowing and represents a certain, contractual obligation. Its sensitivity to monetary policy—through the central bank's policy rate—is direct and strong. Formally, the mortgage rate is modelled as the sum of a risk-free benchmark rate and a debt spread $\rho^m > 0$ reflecting intermediation costs:

$$r_t^m = r_t^{\mathrm{f}} + \rho^m \tag{3.11}$$

where r_t^{f} is the risk-free rate (e.g., the government bond yield or the central bank policy rate).

Equity opportunity cost r_t^e. The equity component of housing finance represents the household's own wealth deployed in the dwelling. The opportunity cost of this equity is the return the household forgoes by *not* investing that wealth in an alternative financial asset—equities, bonds, or a diversified portfolio. Unlike the mortgage rate, this is not a contractual payment but an *implicit* forgone return. Its magnitude reflects the risk premium demanded by investors for holding illiquid residential equity rather than a liquid diversified portfolio. The equity opportunity cost thus carries a *risk premium* ρ^e over the risk-free rate that is structurally larger than the debt spread:

$$r_t^e = r_t^{\mathrm{f}} + \rho^e, \qquad \rho^e > \rho^m > 0 \tag{3.12}$$

The inequality $\rho^e > \rho^m$ reflects the standard asset-pricing result that equity claims, being residual and illiquid, must offer a higher expected return than senior debt claims to attract investors. This ordering, $r_t^e > r_t^m$, holds unconditionally and independently of the level of interest rates or the state of the housing market.

The FUC Model

Let $\lambda \in [0,1]$ denote the *loan-to-value ratio* (LTV), i.e., the fraction of the property value P_t^H financed by mortgage debt. The fraction $(1-\lambda)$ is financed by equity. The *composite financing rate* for a household with LTV ratio λ is the leverage-weighted average of the two rates:

$$r_t^f(\lambda) \;\equiv\; \lambda\, r_t^m \;+\; (1-\lambda)\, r_t^e \tag{3.13}$$

Substituting (3.11) and (3.12) into (3.13) and writing $c \equiv \delta + \tau + m$ for the carrying cost rate, the *Financial User Cost rate* is:

$$u_t^f(\lambda) \;=\; r_t^f(\lambda) + c - \pi_t^e \;=\; r_t^{\mathrm{f}} + \underbrace{\lambda\, \rho^m + (1-\lambda)\, \rho^e}_{\text{financing premium} > 0} + c - \pi_t^e \tag{3.14}$$

(Online Appendix E, equation (E.4)). The FUC generalises the Basic User Cost in two respects. First, it replaces the single interest rate with the composite financing rate $r_t^f(\lambda)$, which depends on the household's leverage position λ. Second, the *financing premium* $\lambda\rho^m + (1-\lambda)\rho^e$ is always strictly positive and monotonically decreasing in λ: higher leverage substitutes cheap debt for expensive equity, lowering the composite rate. This implies the sign ordering:

$$\frac{\partial\, u_t^f(\lambda)}{\partial \lambda} \;=\; \rho^m - \rho^e \;<\; 0 \tag{3.15}$$

More leveraged households face a *lower* user cost, all else equal. This is the channel through which credit-market expansion—an increase in available λ—reduces the effective cost of housing services and stimulates demand.

FUC, Asset Prices, and Bubble Dynamics

The FUC framework provides a direct link between housing finance conditions and the dynamics of asset prices, illuminating the mechanism through which speculative bubbles emerge.

Recall the asset-pricing identity (3.4): the current asset price P_t^H equals the present discounted value of all future user costs. In steady state, this collapses to:

$$P_t^H = \frac{U_t^f}{r_t^f(\lambda) - \pi_t^e + \delta} = \frac{U_t^f}{u_t^f(\lambda) - c + \delta} \tag{3.16}$$

Equation (3.16) encodes the two channels through which credit and expectations drive asset prices.

The credit channel. An expansion of mortgage lending—a rise in λ or a compression of ρ^m—lowers $r_t^f(\lambda)$ via (3.13). From (3.14), this reduces u_t^f, and from (3.16), a lower user cost denominator raises the asset price P_t^H for any given service flow U_t^f. The causal chain is:

$$\uparrow \lambda \ \text{ or } \ \downarrow \rho^m \implies \downarrow r_t^f(\lambda) \implies \downarrow u_t^f \implies \uparrow P_t^H$$

This is the mechanism by which credit expansions fuel housing price booms: easy lending conditions lower the effective cost of holding housing, boosting demand and pushing up valuations.

The expectations channel. An upward revision in π_t^e has the same directional effect via (3.14) and (3.16): it reduces u_t^f and raises P_t^H. Crucially, the resulting capital gain then *validates* the expectation, inducing further upward revisions—a self-reinforcing expectations spiral. The bubble condition is the knife-edge at which the user cost is driven to zero:

$$u_t^f(\lambda) = 0 \iff \pi_t^e = r_t^f(\lambda) + c \tag{3.17}$$

When π_t^e exceeds the threshold $r_t^f(\lambda) + c$, the user cost turns strictly negative: the expected capital gain more than compensates for all costs of holding the asset, so the dwelling "pays" the owner to hold it. In this regime, demand is driven not by the welfare value of shelter services but purely by the anticipated investment return. The market has crossed from the *welfare layer*—where housing is priced by its consumption value—into the *investment layer*, where it is priced by expected capital appreciation.

Credit amplification of the bubble. The two channels interact. When π_t^e rises, u_t^f falls, making housing more attractive to leveraged buyers. Financial institutions, observing rising collateral values, expand lending ($\uparrow \lambda$), which further lowers $r_t^f(\lambda)$ and u_t^f, which in turn validates further price increases. This feedback loop between credit expansion and asset price appreciation is the hallmark of speculative housing bubbles. The bubble condition (3.17) shifts *rightward* (i.e., is reached at a lower level of π_t^e) as leverage increases, because higher λ lowers $r_t^f(\lambda)$. Formally, differentiating (3.17) with respect to λ:

$$\frac{\partial \pi_t^{e,*}}{\partial \lambda} = \frac{\partial r_t^f(\lambda)}{\partial \lambda} = \rho^m - \rho^e < 0 \tag{3.18}$$

Higher leverage *lowers* the expected-appreciation threshold at which the bubble condition is triggered: credit expansion makes it easier for the market to enter the investment layer.

The market crosses from the *welfare layer*—where housing is priced by its consumption value—into the *investment layer*, where it is priced by expected capital appreciation. The theoretical and measurement consequences of this regime transition, and the opportunity cost approach that bridges the two layers, are developed in Section 3.4.

3.4 The Dual Nature of Housing and the Opportunity Cost Approach

Welfare Layer and Investment Layer

The theoretical framework of Sections 3.1–3.3 reveals that housing simultaneously occupies two conceptually distinct economic layers whose divergence generates the observed disconnect between rental prices and asset prices.

The *welfare layer* is the layer in which housing is valued as a consumption good. A household occupying a dwelling derives a flow of shelter services—warmth, space, security, locational access—whose shadow price is determined by preferences, income, and available alternatives. In this layer, the relevant price is the minimum per-period cost of obtaining a constant-quality unit of shelter, which is precisely what the Könus COLI requires. The rental equivalence approach (with new-contract rents) and the user cost approach both seek to measure this welfare-layer price. The welfare-layer price responds to changes in household preferences, population, and the physical stock of housing, but not to financial conditions *per se*.

The *investment layer* is the layer in which housing is valued as a financial asset. A dwelling is a claim on a future stream of shelter services, and its market price is the present discounted value of all future service flows, capitalised at the household's opportunity cost of capital net of expected appreciation. In this layer, the relevant price is the asset price P_t^H, which responds to changes in interest rates, credit conditions, and expectations about future prices. The acquisitions approach operates entirely in the investment layer, measuring asset price movements rather than service-flow costs.

The conceptual distance between the two approaches is therefore not merely quantitative but qualitative. The rental equivalence approach asks: "what does the market charge to rent this dwelling for one period?"—a welfare-layer question. The user cost approach asks: "what is the net financial cost of owning this dwelling for one period?"—a question that bridges the two layers through the user cost formula (3.14), decomposing the asset's holding cost into a welfare component ($r_t^f + c$) and an investment component ($-\pi_t^e$). The acquisitions approach asks: "what is the market price of this dwelling?"—a pure investment-layer question.

In a frictionless market in long-run equilibrium, the two layers are connected by the landlord's zero-profit condition (Online Appendix B, equation (B.4)): equilibrium rent equals user cost, which in turn reflects the capitalisation rate applied to the asset price. This connection breaks down precisely when it matters most: during episodes of rapid asset price appreciation or contraction, the investment layer dominates, the asset price diverges from its fundamental value, and the welfare-layer price of shelter moves in the *opposite direction* to the investment-layer price. Formally, differentiating the asset-pricing identity (3.4) with respect to π_t^e yields:

$$\frac{\partial P_t^H}{\partial \pi_t^e} > 0, \qquad \frac{\partial u_t^f}{\partial \pi_t^e} = -1 < 0 \tag{3.19}$$

A rise in expected capital gains simultaneously *raises* the asset price and *lowers* the welfare-layer cost of owner-occupancy. This sign reversal is the mathematical signature of the welfare–investment layer divergence.

The Opportunity Cost Approach

The COLI measures the minimum expenditure required for a household to achieve a given utility level; it is therefore a welfare concept. The COLI-consistent period price of housing services is the welfare-layer price—the minimum cost of obtaining one unit of shelter in the current period, given the best available alternatives.

In a market without frictions, a utility-maximising household chooses between renting at the new-contract rent R_t^N or owning at the Financial User Cost $\text{FUC}_t(\lambda)$. The COLI-consistent price is the *minimum* of the two:

$$p_{\text{COLI}}^{h,t} \;=\; \min\bigl\{\text{FUC}_t(\lambda)\cdot P_t^H,\; R_t^N\bigr\} \tag{3.20}$$

(Online Appendix B, equation (B.2)). In competitive equilibrium, $\text{FUC}_t \cdot P_t^H = R_t^N$, and (3.20) reduces to either approach. In disequilibrium, the household chooses the cheaper mode of access, and the COLI price is the lower of the two.

Diewert (2008)[7] observes, however, that a homeowner foregoes the *maximum* of the two alternatives by not switching tenure. The owner-occupier's true opportunity cost is therefore:

$$\text{OC}_t(\lambda) \;=\; \max\left\{\text{FUC}_t(\lambda) \cdot P_t^H,\; R_t^N\right\} \tag{3.21}$$

The relationship between the OC approach and the COLI depends on the prevailing market regime. When $\text{FUC}_t \cdot P_t^H < R_t^N$ (the *investment regime*: capital gains are large, user cost is low), both the COLI (3.20) and the OC (3.21) equal R_t^N—the two coincide. When $\text{FUC}_t \cdot P_t^H > R_t^N$ (the *capital-cost regime*: high interest rates, low expected appreciation), the COLI equals R_t^N while the OC equals $\text{FUC}_t \cdot P_t^H$: the OC provides an upper bound on the true COLI price. This regime analysis is summarised in the ordering above.

The OC approach is always non-negative—it avoids the implausible negative prices that afflict the user cost approach during bubble episodes—and provides an upper bound on both the rental equivalence and user cost measures individually. These properties make it a useful benchmark for assessing the sensitivity of CPI measurement across different market regimes.

Why Negative User Costs Are Welfare-Theoretically Inadmissible

A natural question arises: if a household genuinely benefits from owning a dwelling during a bubble episode—because expected capital gains exceed all carrying costs—is not the user cost correctly negative, reflecting the true economic gain? This objection, while intuitive in the context of investment accounting, rests on a conflation of two distinct economic roles of housing that the COLI framework is specifically designed to separate.

The COLI measures the minimum expenditure required to achieve a given *utility level*, not a given level of financial wealth. Shelter services—warmth, space, security, locational access—enter the household utility function directly and independently of the asset's financial returns. A household's utility from occupying a dwelling for one period is strictly positive by definition: the dwelling delivers a positive flow of shelter services. The COLI-consistent price of that flow must therefore be strictly non-negative, because no household would willingly forgo a positive utility flow in exchange for a zero payment.

Formally, the household's problem can be written as $\min_{\mathbf{q}} \mathbf{p} \cdot \mathbf{q}$ subject to $U(\mathbf{q}) \geq \bar{u}$, where housing services $h \in \mathbf{q}$ enter U with a strictly positive marginal utility $\partial U/\partial h > 0$. The Lagrangian conditions require that the COLI price of housing services, $p^h = \lambda \cdot (\partial U/\partial h) > 0$, where $\lambda > 0$ is the expenditure multiplier. A negative price for a good with positive marginal utility is inconsistent with utility maximisation: it would imply that the household should consume an infinite quantity of shelter services, which violates feasibility. The COLI therefore *cannot* assign a negative price to housing services, regardless of the financial returns accruing to the asset. The negative values generated by the UC formula during bubbles reflect the conflation of the welfare layer and the investment layer: the term $-\pi_t^e$ captures a financial benefit to the asset holder, not a reduction in the consumption cost of shelter.

The OC approach resolves this conflation by construction. In the investment regime ($\text{FUC}_t \cdot P_t^H < R_t^N$), the dwelling's financial advantage is real, but the household's consumption of shelter services costs at least R_t^N—the competitive market price at which the same shelter services can be obtained from rental. No utility-maximising household would value its shelter flow at less than R_t^N if it is the opportunity cost of the best alternative. Setting $\text{OC}_t = R_t^N$ in this regime is not a statistical adjustment; it is the application of the opportunity cost principle of the COLI itself. The financial gain from holding the asset above and beyond this consumption cost belongs to the household's *investment account*, not its *consumption account*, and should be recorded accordingly in asset price indices or balance sheets—not in the CPI.

A fundamental requirement of the COLI is that the period price $p^{h,t}$ refer to a *constant-quality* unit of housing services. If the quality of the dwelling changes between periods—as it does continuously through physical deterioration of the structure—then a comparison of observed prices across periods conflates genuine price changes with quality changes, producing a biased index. This is the *depreciation measurement problem*, and it is central to both the rental equivalence and user cost approaches.

To see the problem formally, let q_t denote the quality-adjusted quantity of housing services delivered by a dwelling of age A in period t. Under geometric depreciation at rate δ:

$$q_t = (1-\delta)^A q_0 \tag{3.22}$$

where q_0 is the service flow at age zero. The COLI requires the price of one *constant-quality* unit, i.e., the observed price normalised by the quality index:

$$p^{h,t}_{\text{const-qual}} = \frac{p^{h,t}_{\text{observed}}}{(1-\delta)^A} \tag{3.23}$$

If depreciation is ignored ($\delta = 0$), the price index declines over time simply because older dwellings deliver lower-quality services, even if the true constant-quality price is rising. As Japan's housing stock ages—the average age of dwellings has risen steadily over the study period—a CPI that ignores depreciation will systematically *understate* the true cost of constant-quality shelter services.

This bias has distinct manifestations in the two approaches. In the rental equivalence approach, the observed rent for a dwelling of age A reflects the market valuation of $(1-\delta)^A$ units of constant-quality service; the COLI-consistent rent per unit is $R_t(A)/(1-\delta)^A$, which rises with age. A rent survey that does not quality-adjust for building age will understate the true price increase as the stock ages. In the user cost approach, δ enters the user cost formula directly as the rate at which the service flow declines; an underestimate of δ directly underestimates the carrying cost and thus the service-flow price.

The COLI requirement of constant quality therefore demands that δ be estimated consistently across the rental and ownership sectors, and that identical age adjustments be applied in both. Inconsistent depreciation assumptions introduce systematic wedges between the rental equivalence and user cost approaches that reflect measurement choices rather than market conditions. The depreciation estimation methodology used in this paper, based on hedonic regressions on the full 40-year transaction dataset, is described in Online Appendix C.

Long-Run Expectations and the Service-Flow Price

The user cost formula (3.14) contains the *expected* capital gain rate π_t^e, which is unobservable. The choice of proxy for this expectation is not merely technical: it is a substantive question about the *horizon* at which households form expectations when making housing decisions, with direct consequences for the COLI-consistency of the resulting price index.

Housing is a long-lived asset whose service life typically exceeds fifty years. A household purchasing a dwelling makes a decision whose welfare implications extend over decades. The expectations relevant to the user cost formula are therefore *long-run expectations* about the secular trend in house prices, not short-run speculative forecasts. Using the short-run realised appreciation π_t^{act} as a proxy for π_t^e is conceptually inappropriate for the COLI, because it introduces investment-layer volatility into the welfare-layer price of shelter—precisely the conflation the COLI framework is designed to avoid.

Formally, let π^L denote the long-run secular appreciation rate, reflecting fundamental drivers such as population growth, income growth, and land supply constraints. A household with rational long-run expectations sets $\pi_t^e = \pi^L$. The COLI-consistent user cost is then:

$$u_t^{f,L}(\lambda) \;=\; r_t^f(\lambda) + c - \pi^L \tag{3.24}$$

Equation (3.24) varies over time only through changes in $r_t^f(\lambda)$, which responds to monetary policy and credit conditions, but not to short-run speculative movements in asset prices. This is the appropriate welfare-layer measure.

The deviation of realised appreciation from the long-run rate defines the *investment premium* $\phi_t \equiv \pi_t^{\text{act}} - \pi^L$. Using the realised rate instead of the long-run rate introduces $-\phi_t$ into the measured user cost:

$$u_t^{f,\text{act}} - u_t^{f,L} \;=\; -(\pi_t^{\text{act}} - \pi^L) \;=\; -\phi_t \tag{3.25}$$

During booms ($\phi_t > 0$), the realised-appreciation user cost is *lower* than the welfare-layer user cost by the investment premium; during busts ($\phi_t < 0$), it is *higher*. This is the mechanism by which short-run expectation proxies import investment-layer volatility into the measured COLI, generating the procyclical biases documented in Section 3.1. In this paper, π^L is estimated from the long-run HP-filtered trend of the 40-year appreciation series (Online Appendix A).

Household Heterogeneity and the Limits of Aggregate Measurement

A comprehensive welfare analysis of housing costs must acknowledge a fundamental distributional asymmetry that is invisible in aggregate price indices: the cost of housing services differs radically across households depending on whether they own or rent, when they entered the market, and how they are financed.

In most advanced economies—and Japan in particular—housing wealth is concentrated among older and wealthier households who purchased their dwellings decades ago at much lower prices. For these households, the nominal user cost is anchored to the historical purchase price P_τ^H ($\tau \ll t$) and may be very low; moreover, the unrealised capital gain $P_t^H - P_\tau^H$ further augments their welfare. Younger and lower-income households who do not yet own are subject to the full impact of rising prices: they either pay current market rents or face the prospect of purchasing at elevated prices and bearing the full financing cost at current mortgage rates.

This distributional asymmetry can be formalised through the COLI. For a homeowner who purchased at price P_τ^H in period $\tau < t$, the welfare-layer cost in period t is approximately $u_t^f(\lambda) \cdot P_\tau^H$. For a new buyer, it is $u_t^f(\lambda) \cdot P_t^H$. The ratio of the two COLI prices is:

$$\frac{p_{\text{new buyer}}^{h,t}}{p_{\text{incumbent}}^{h,t}} \approx \frac{P_t^H}{P_\tau^H} = \prod_{s=\tau}^{t-1}(1+\pi_s^{\text{act}}) \gg 1 \tag{3.26}$$

during sustained appreciation. This generational heterogeneity in housing costs—between those who already own and those who do not—has grown substantially over the 40-year study period for Tokyo. A fully satisfactory treatment would require a *heterogeneous-agent* extension of the COLI framework, computing separate sub-indices by age, income, tenure status, and entry cohort, as explored theoretically by Diewert and Nakamura (2011)[9]. Such an extension requires household-level data on purchase prices, financing conditions, and tenure history that are rarely available at the scale and frequency required for CPI compilation. This paper does not pursue the heterogeneous-agent framework; the empirical analysis in Section 4 is conducted at the aggregate level. The heterogeneity issue is, however, an important caveat to the interpretation of the aggregate index numbers presented here, and remains a central open problem for future research.

4 Empirical Comparison of OOH Measurement Approaches

4.1 Data and Quality-Adjusted Price and Rent Indices

The empirical analysis draws on individual property listing records compiled by Recruit Co., Ltd. from the SUUMO platform, the largest real estate information service in Japan. The underlying Recruit/SUUMO database spans the Greater Tokyo Area (Tokyo Metropolis, Kanagawa, Saitama, and Chiba prefectures), but the present study restricts the hedonic estimation and all index construction to the **Tokyo Special District (the 23 wards)**, the densest and most liquid segment of the Tokyo condominium market and the geographic unit most directly comparable to the official Tokyo CPI OOH component. The 23-ward sample comprises approximately 357,627 condominium sale records, 615,791 single-family house sale records, and 2,139,043 new-lease condominium rental contracts over the period January 1986 to December 2025—480 monthly observations in total. A detailed description of the data and hedonic methodology is provided in Appendix F (Tables F.8–F.10 and Figure F.6); this section summarises the key features relevant to the measurement comparison.

The Recruit/SUUMO records are listing-based: each observation records the asking price posted by the seller at the time of advertisement, not the final negotiated transaction price. A natural concern is whether asking prices are systematically biased relative to transaction prices, which could introduce an upward-level bias into the quality-adjusted indices. Shimizu, Nishimura and Watanabe (2016)[17] directly address this concern by comparing the full cross-sectional *distributions* of four price types collected at successive stages of the buying/selling process for Tokyo condominiums: (1) initial asking prices from SUUMO listings, (2) final asking prices at the time of offer, (3) contract prices from the REINS realtor database, and (4) registry prices from the Ministry of Land, Infrastructure, Transport and Tourism. Using quantile hedonic regressions and Kolmogorov–Smirnov goodness-of-fit tests, they find that the unconditional distributions differ substantially—mainly because the four datasets cover different sub-populations of dwellings—but that once quality differences are controlled for, "only small differences remain between the different house price distributions." The KS statistic for quality-adjusted initial asking prices versus registry prices is $D = 0.058$, which, while statistically significant, is economically small relative to the magnitude of the cross-regime divergences documented in this paper. These results establish that asking-price-based hedonic indices produce quality-adjusted measures that are *comparable* to transaction-price indices, provided appropriate quality controls are applied—precisely the methodology employed in Appendix F.

The quality-adjusted price index $P^H(t)$ and new-lease rent index $R^{\text{new}}(t)$ are constructed by the rolling-window hedonic methodology of Shimizu, Nishimura and Watanabe (2010a)[41] (Appendix F, Sections F.2–F.3; see also Section F.7 on advantages over repeat-sales), evaluating the fitted hedonic surface at fixed standard characteristics: $60\,\text{m}^2$ floor space, 10-year building age, 5-minute walk to the nearest station, 20-minute commute to Otemachi CBD, Shinjuku Ward, reinforced concrete structure. Both indices are normalised so that their 2020 annual mean equals 100. The official CPI series used as the benchmark in Section 4.4 is the Tokyo CPI OOH component (imputed rent) published monthly by the Statistics Bureau of Japan, also normalised to 2020 = 100.

Table 3 summarises the key series over the full sample and across the four market episodes defined in the calibration (Appendix E): the Bubble (1987:01–1990:12), Deflation (1993:01–2003:12), Low-rate (2014:01–2020:12), and Recent (2021:01–2025:12) episodes.

Table 3: Summary Statistics: Quality-Adjusted Price, Rent, and CPI OOH by Market Episode (2020 = 100)

Series	Bubble	Deflation	Low-rate	Recent	Full
$P^H(t)$ (price index)	118.0	69.1	91.5	117.6	85.9
$R^{\text{new}}(t)$ (rent)	87.7	89.0	94.8	104.8	92.8
Official CPI OOH	90.8	106.4	99.9	100.8	101.2
Rent-to-price ratio R/P (%)	1.9	3.5	2.6	2.3	3.0
HP-smoothed π^e (% p.a.)	14.9	−7.1	4.6	5.3	2.1
BOJ prime rate (% p.a.)	6.2	2.8	1.0	1.5	2.7

Note: Episode-mean values. Price and rent indices are normalised to 2020 = 100. π^e is the HP-filtered expected appreciation rate ($\bar{\lambda}_{\text{HP}} = 129{,}600$). Full-sample: 480 monthly observations, 1986:01–2025:12.

Several features of the data are immediately noteworthy. The price index exhibits three large cycles over the 40-year sample, ranging from a Bubble-era mean of 118.0 to a Deflation-era trough of 69.1—a peak-to-trough decline of approximately 41 per cent. The rent index, by contrast, is strikingly stable: it ranges from 87.7 (Bubble) to 104.8 (Recent), a full-sample coefficient of variation far below that of the price index. This divergence between price volatility and rent stability is the central empirical manifestation of the dual nature of housing documented in Section 3: the investment layer drives the asset price, while the welfare layer anchors the flow cost of shelter. The HP-smoothed expected appreciation π^e ranges from +14.9 per cent per annum in the Bubble to −7.1 per cent in the Deflation, a swing of more than 22 percentage points that is the primary driver of the measurement divergence quantified in the following subsections.

4.2 Construction of the Five Measurement Series

All five measurement series are constructed from the common dataset described in Section 4.1, using the calibration parameters of Appendix E (Table E.6). This section summarises the construction of each series; full computational details are given in Appendix A.

Acquisitions approach. The acquisitions index is the quality-adjusted price index $P^H(t)$ itself, normalised to 2020 = 100. Its annual inflation rate is the first log-difference $\Delta \ln P^H(t) \times 12$, which equals the annualised realised appreciation π_t^{act}. No imputation or parameter calibration is required beyond the rolling-window hedonic regression (Appendix F; see also Table F.8 and Table F.9).

Basic User Cost (HP-smoothed). The Basic UC rate is $u_t^{\text{basic}} = r_t^{\text{prime}} + c - \pi_t^e$, where $c = \delta + \tau + m = 0.034$ is the carrying cost rate and π_t^e is the HP-filtered expected appreciation with $\bar{\lambda}_{\text{HP}} = 129{,}600$ (robustness analysis in Appendix G, Section G.1). The monetary index is $U_t^{\text{basic}} = u_t^{\text{basic}} \times P^H(t)$, normalised to 2020 = 100. We use the HP-smoothed variant throughout the main text; the realised-appreciation variant (Basic UC act) is volatile and produces negative values in 70.8 per cent of Bubble-episode months, making it unsuitable as a standalone CPI measure. The depreciation rate $\delta = 0.020$ is calibrated from the hedonic estimates described in Appendix C (Section C.3 and Table C.7): the pooled log-linear estimate yields $\hat{\delta} = 0.0204$ per annum (SE = 0.0003), and the rolling-window mean is

0.0203—both within a rounding margin of the MLIT benchmark of 0.020 (see also [30][31]). The sensitivity of the FUC to this calibration is documented in Table G.16 (Appendix G, Section G.7): a ± 0.2 pp band around $\delta = 0.020$ shifts the user cost rate by exactly ± 0.002 per annum in every observation.

Financial User Cost: Types A, B, and C. The FUC framework (Appendix E) replaces the single prime rate with a leverage-weighted composite financing rate $r_t^f(\lambda) = \lambda r_t^m + (1 - \lambda) r_t^e$, where $r_t^m = r_t^{\text{prime}} + 0.005$ is the mortgage rate and $r_t^e = r_t^{\text{prime}} + 0.020$ is the equity opportunity cost rate. Three household types are defined by their LTV ratio λ: Type A ($\lambda = 0.20$, low-leverage), Type B ($\lambda = 0.50$, baseline), and Type C ($\lambda = 0.80$, high-leverage). The FUC rate and monetary index for type j are:

$$u_t^j = r_t^f(\lambda_j) + c - \pi_t^e, \quad U_t^j = u_t^j \times P^H(t), \quad j \in \{A, B, C\} \tag{4.1}$$

The FUC exceeds the Basic UC by a constant financing premium: $u_t^j - u_t^{\text{basic}} = \lambda_j \rho^m + (1 - \lambda_j)\rho^e \in \{0.017,\ 0.0125,\ 0.008\}$ for Types A, B, C respectively (see equation E.13).

Payments approach. Following the framework of Appendix A.4 and Goodhart (2001)[11], the payments rate captures the cash outflows borne by owner-occupiers from mortgage interest and property tax:

$$u_t^{\text{pay},j} = \lambda_j\, r_t^m + \tau, \quad U_t^{\text{pay},j} = u_t^{\text{pay},j} \times P^H(t) \tag{4.2}$$

Principal repayment is excluded as a saving component. Three variants are constructed corresponding to the same leverage types A, B, C. Unlike the FUC, the payments rate is strictly positive in all periods: it rises when nominal mortgage rates rise—regardless of whether expected appreciation is offsetting the carrying cost—and therefore does not capture the true economic cost of shelter.

New-contract rent index. The quality-adjusted new-lease rent index $R^{\text{new}}(t)$, normalised to $2020 = 100$, serves as both the rental-equivalence measure and the COLI-consistent benchmark in the opportunity cost computation. It is constructed from the same rolling-window hedonic as the price index, using the rental sub-sample of 2,139,043 new-lease contracts.

4.3 Comparison of Index Levels and Inflation Rates

Table G.11 (Appendix G, Section G.2) presents the episode-mean levels of all measurement series, normalised to $2020 = 100$. The variation across approaches and episodes is substantial and follows the theoretical predictions of Section 3 closely (Figure 1).

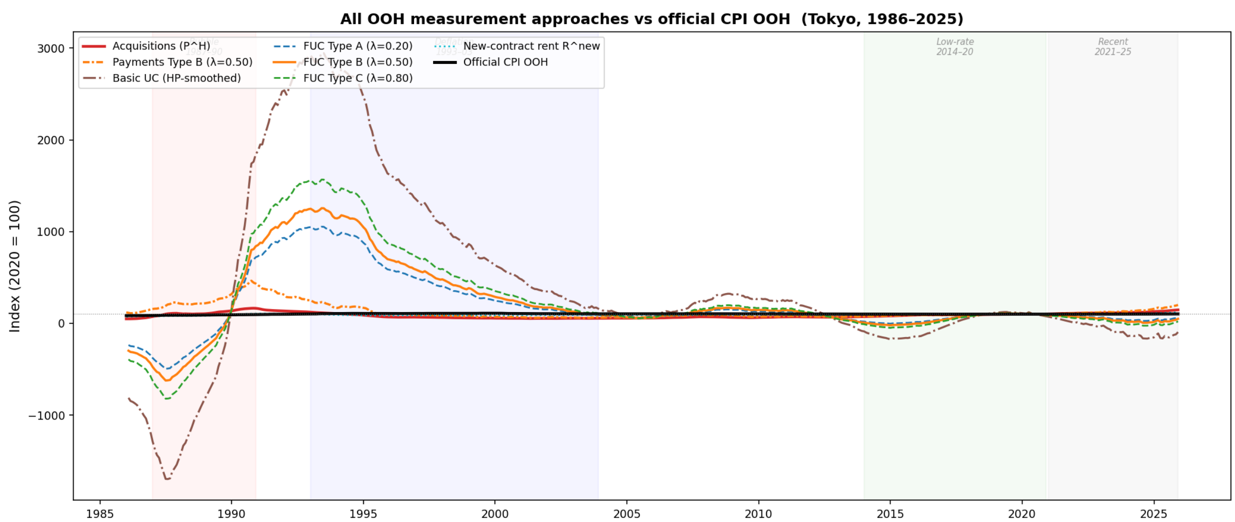

Figure 1: All OOH measurement approaches and official CPI OOH, Tokyo 1986–2025 ($2020 = 100$). Shaded regions denote the Bubble (red), Deflation (blue), Low-rate (green), and Recent (grey) episodes.

The range of measured values across approaches is extraordinary. During the Bubble episode, the acquisitions index stood at 118.0 while the FUC Type B index was −143.7—a gap of 261.7 index points. During the Deflation episode, the FUC Type B index reached 532.9 while the acquisitions index fell to 69.1—a gap of 463.8 points. These divergences reflect the sign reversal documented in equation (3.19): the acquisitions index tracks asset prices ($\partial P^H/\partial \pi^e > 0$), while the FUC tracks the welfare-layer service cost ($\partial u^f/\partial \pi^e = -1$), so the two measures move in *opposite directions* when expected appreciation changes.

Table 4: Annual User Cost Rates (% p.a.) by Approach and Episode

Approach	Bubble	Deflation	Low-rate	Recent	Full
Rent-to-price (R/P)	1.92	3.45	2.63	2.26	2.97
Acquisitions ($\Delta \ln P^H$)	21.56	−7.35	4.81	7.28	2.76
Basic UC (HP-smoothed)	−5.31	13.32	−0.18	−0.43	4.02
Payments A ($\lambda = 0.20$)	2.73	2.06	1.71	1.80	2.04
Payments B ($\lambda = 0.50$)	4.73	3.04	2.17	2.40	3.00
Payments C ($\lambda = 0.80$)	6.73	4.02	2.63	3.01	3.96
FUC Type A ($\lambda = 0.20$)	−3.61	15.02	1.52	1.27	5.72
FUC Type B ($\lambda = 0.50$)	−4.06	14.57	1.07	0.82	5.27
FUC Type C ($\lambda = 0.80$)	−4.51	14.12	0.62	0.37	4.82

Note: Acquisitions rate $= \Delta \ln P^H \times 12$ (annualised). Basic UC and FUC use HP-smoothed π^e; payments rate $= \lambda r^m + \tau$.

Table 4 reports the annual user cost rates. The contrast between the Bubble and Deflation episodes is striking. During the Bubble, the HP-smoothed expected appreciation of 14.9 per cent per annum far exceeded the prime rate (6.2%) plus the carrying cost (3.4%), driving the Basic UC to −5.31% and the FUC types to between −3.61% (Type A) and −4.51% (Type C). The FUC turned negative in 68.8–72.9 per cent of Bubble-episode months depending on the leverage type (Table G.12; Appendix G, Section G.3). The acquisitions rate, by contrast, averaged +21.56% per annum—signalling severe inflation at the precise moment when the true per-period cost of shelter was at its historical minimum.

During the Deflation, the pattern reversed completely. With the prime rate at 2.78% and expected appreciation at −7.14% per annum, the Basic UC rose to 13.32% and the FUC rates to between 14.12% (Type C) and 15.02% (Type A)—more than five times their full-sample averages. The payments approach, in contrast, moved modestly: from a Bubble-era rate of 4.73% (Type B) to 3.04% during the Deflation, tracking the modest decline in nominal mortgage rates. This illustrates the fundamental limitation of the payments approach identified in Section 3.1: by ignoring expected capital gains, it fails to capture the massive swing in the true economic cost of owner-occupancy across regimes.

4.4 Divergence from the Official CPI

Table 6 reports the mean divergence of each alternative OOH measure from the official CPI OOH component, in index points (2020 = 100). The sign of the divergence—whether each alternative is above or below the official CPI—changes systematically across episodes, and the magnitudes are large enough to be of direct relevance for monetary policy (Figure 2).

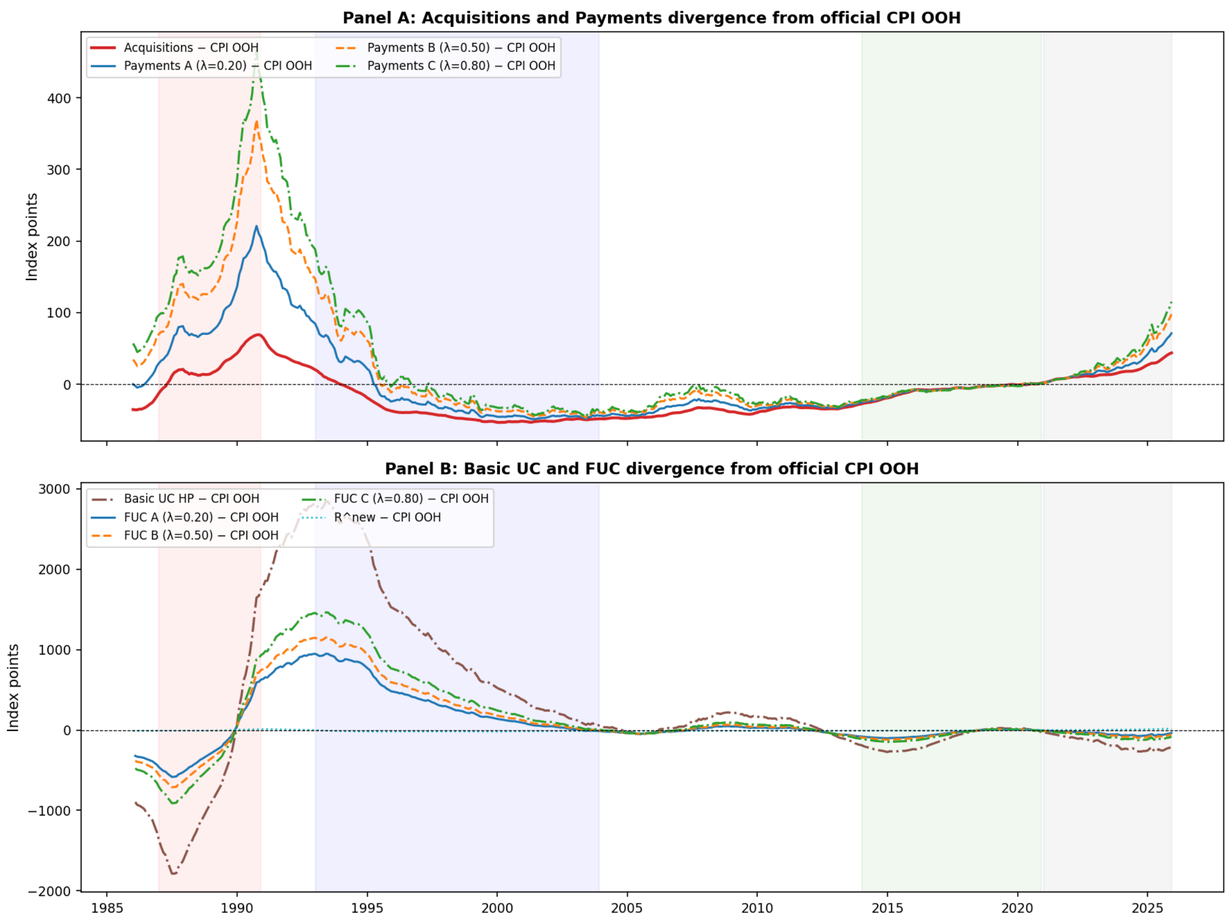

Figure 2: Divergence from the official CPI OOH component (index points, 2020 = 100). Panel A: Acquisitions and Payments approaches A, B, C. Panel B: Basic UC (HP-smoothed) and FUC Types A, B, C.

Four patterns stand out.

First, the user cost approaches (Basic UC and FUC) are uniformly *below* the official CPI during the Bubble and the two post-2013 episodes, and *above* the official CPI during the Deflation. This counter-cyclical pattern is theoretically predicted: when expected appreciation is high, the user cost is compressed below any sticky-rent index; when prices are falling, the user cost surges above it. The FUC Type B divergence ranges from −234.5 points (Bubble) to +426.5 points (Deflation)—a swing of 661 index points over the 40-year sample.

Second, the acquisitions approach and the payments approach are *pro-cyclical* relative to the official CPI: both are above the CPI during asset-price appreciations (Bubble: +27.2 and +174.1 for acquisitions and Payments B respectively) and below during deflation (−37.3 and −4.3). The payments approach is especially sensitive to nominal interest rate movements: during the Bubble, when the prime rate reached 6.2% per annum, Payments B stood 174.1 points above the official CPI— not because housing services had become more expensive in welfare terms but because nominal mortgage payments had risen.

Third, the new-contract rent index R^{new} is the most stable alternative. Its divergence from the official CPI is modest in absolute magnitude (ranging from −17.4 to +4.0 points) and switches sign appropriately: the official CPI overstated market rents during the Deflation (when new-contract rents fell but incumbent rents were sticky) and began understating them in the Recent episode as post-pandemic appreciation outpaced the official index.

Fourth, over the full 40-year sample, the FUC approaches all lie *above* the official CPI (by 94.0–153.8 points depending on leverage type), while the acquisitions index and new-contract rent lie *below* (−15.3 and −8.4 points respectively). This ordering—Basic UC and FUC above CPI, acquisitions and rent below—holds robustly across alternative parameter calibrations and is consistent with the theoretical prediction that the user cost approach, which incorporates the full equity opportunity cost, will on average exceed a sticky-rent CPI that largely tracks historical conditions.

4.5 The FUC Financing-Structure Spread: Type A vs. Type B vs. Type C

A distinctive feature of the FUC framework is the *constant financing-structure spread* between leverage types. From Appendix E (equation E.12), the spread between Type A and Type C is:

$$u_t^A - u_t^C = (\lambda_A - \lambda_C)(\rho^e - \rho^m) = (0.20 - 0.80)(0.020 - 0.005) = 0.90 \text{ pp} \tag{4.3}$$

and the spread between adjacent types is $u_t^A - u_t^B = u_t^B - u_t^C = 0.45$ pp. These spreads are *independent of the prime rate and of expected capital gains*: they depend only on the difference in leverage ratios and on the fixed premium parameters ρ^m and ρ^e. The empirical data confirm this prediction exactly: the estimated spread $u_t^A - u_t^C$ has a full-sample mean of 0.9000 pp with a standard deviation of zero (to eight decimal places), confirming that the spread is strictly constant throughout the 480-month sample (Figure G.11 in Appendix G, Section G.4 plots the three FUC rate series and the constancy of the spread over the full sample).

Table G.13 (Appendix G, Section G.4) documents the mean FUC rates by type and episode, alongside the full-sample composite financing rates from which the spread is derived.

The constancy of the spread has two important empirical implications. First, the *ranking* of FUC types is invariant to the market regime: Type A always exceeds Type B, which always exceeds Type C, regardless of the level of interest rates or the magnitude of expected appreciation. Higher leverage (higher λ) substitutes cheaper mortgage debt for the more expensive equity return, uniformly reducing the FUC rate. Second, the *level* of all three FUC rates moves in parallel: when the Bubble drives expected appreciation upward by x percentage points, all three FUC rates fall by exactly x points; when deflation pushes expected appreciation down by y points, all three rise by y points. The spread therefore provides no information about the phase of the housing cycle—it is a pure *household-heterogeneity* measure, capturing how different financing structures translate into different shelter costs for households at different positions in the wealth and credit-access distribution.

The full-sample mean composite financing rates are 4.39% (Type A), 3.94% (Type B), and 3.49% (Type C), consistently above the mean prime rate of 2.69% by the respective financing premiums of 1.70, 1.25, and 0.80 percentage points. These financing premiums—reflecting the illiquidity premium on residential equity ($\rho^e = 2.0\%$) and the bank intermediation spread on mortgage debt ($\rho^m = 0.5\%$)—imply that even households with identical housing price and rent exposure face materially different ownership costs depending on how they finance their dwelling. Over the 40-year Tokyo sample, this household-level heterogeneity amounts to a persistent full-sample spread of 0.90 percentage points—approximately 30 per cent of the mean rent-to-price ratio of 2.97%—between the least and most leveraged household types.

5 Analytical Framework: The Negative User Cost Problem, Regime Analysis, and Welfare Implications

5.1 The Financial User Cost and the Negative Value Problem

This subsection summarises the Financial User Cost (FUC) framework (Appendix E) and provides a systematic empirical diagnosis of the *negative user cost problem*—the most persistent practical obstacle to the adoption of user cost approaches in official statistics—together with its resolution via the opportunity cost (OC) approach.

The FUC framework

The Basic User Cost uses the prime lending rate as the single opportunity cost of capital, ignoring that households finance dwellings through both mortgage debt (rate $r_t^m = r_t^{\text{prime}} +$

ρ^m, $\rho^m = 0.005$) and own equity (rate $r_t^e = r_t^{\text{prime}} + \rho^e$, $\rho^e = 0.020$). For a household with loan-to-value ratio λ, the composite financing rate and FUC rate are (Appendix E, equations E.3–E.4):

$$r_t^f(\lambda) = \lambda r_t^m + (1-\lambda) r_t^e, \qquad u_t^f(\lambda) = r_t^f(\lambda) + c - \pi_t^e \tag{5.1}$$

where $c = \delta + \tau + m = 0.034$ is the carrying cost and π_t^e is the HP-smoothed expected appreciation (equation 5.1). Three household types are calibrated by LTV ratio: Type A ($\lambda = 0.20$, financing premium +2.00 pp), Type B ($\lambda = 0.50$, +1.25 pp), and Type C ($\lambda = 0.80$, +0.50 pp). The FUC strictly exceeds the Basic UC and preserves the ordering $u_t^C < u_t^B < u_t^A$ with a constant spread of 0.90 pp between the extreme types, independent of the interest rate or appreciation cycle (Appendix E, equations E.11–E.12).

Diagnosis: the negative user cost problem in Tokyo data

The FUC turns negative whenever expected appreciation exceeds the financing cost plus carrying cost: $u_t^f(\lambda) < 0 \Leftrightarrow \pi_t^e > r_t^f(\lambda) + c$. Table 5 documents the incidence of negative values across all user cost variants and all four market episodes.

Table 5: Incidence of Negative User Cost Values by Approach and Episode (% of months)

Approach	Bubble	Deflation	Low-rate	Recent	Full
Basic UC (realised π^{act})	68.8	6.8	46.4	60.0	36.2
Basic UC (HP-smoothed π^e)	72.9	0.0	48.8	75.0	29.4
FUC Type A ($\lambda = 0.20$)	68.8	0.0	4.8	0.0	10.0
FUC Type B ($\lambda = 0.50$)	70.8	0.0	28.6	0.0	14.4
FUC Type C ($\lambda = 0.80$)	72.9	0.0	38.1	35.0	21.0
OC Type A ($= \max\{\text{FUC}_A \cdot P^H,\ R^N\}$)	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**
OC Type B	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**
OC Type C	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**

Note: $N = 480$ months total (Bubble 48, Deflation 132, Low-rate 84, Recent 60). Basic UC (realised) uses $\pi^{\text{act}} = \Delta \ln P^H \times 12$. FUC and OC use HP-smoothed π^e ($\bar{\lambda}_{\text{HP}} = 129{,}600$). OC $= \max\{\text{FUC} \cdot P^H,\ R^N\}$, which is strictly positive since $R^N > 0$ always.

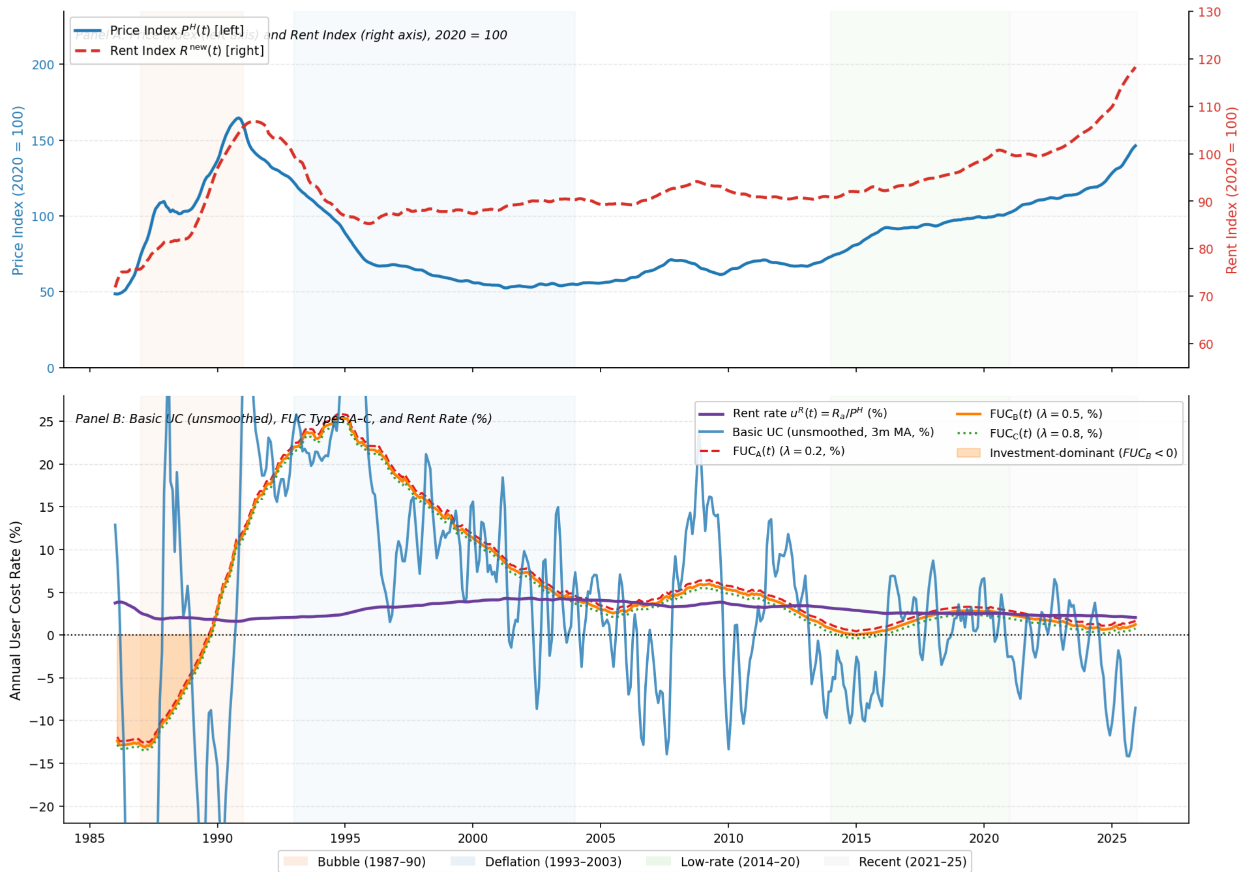

Figure 3: Basic User Cost and Financial User Cost rates (% p.a.) with negative-value episodes highlighted, Tokyo 1986–2025. The FUC turns negative whenever HP-smoothed expected appreciation π_t^e exceeds the financing-cost threshold $r_t^f(\lambda)+c$ (grey shading). The new-contract rent-to-price ratio R^N/P^H (right axis, dotted) is always positive and serves as the floor of the OC approach, guaranteeing a non-negative OC index in every period.

Figure 3 plots the FUC rates with negative-value episodes shaded and the rent-to-price floor superimposed. The severity of the problem is striking. The Basic UC computed with realised appreciation is negative in *36.2 per cent* of all months—174 out of 480—with a minimum value of −73.2 per cent per annum during the Bubble. Even the HP-smoothed Basic UC is negative in 29.4 per cent of months and in 72.9 per cent of Bubble-episode months. The FUC variants improve the situation by raising the threshold $\pi_t^{e,*}(\lambda) = r_t^f(\lambda) + c$: for Type B with a Bubble-era prime rate averaging 6.2 per cent, this threshold is approximately 10.8 per cent, which was nonetheless exceeded by the HP-smoothed appreciation of 14.9 per cent. FUC Type B was negative for 34 *consecutive* months from January 1987 to October 1989, with a minimum of −13.7 per cent (Figure G.9 in Appendix G, Section G.1 plots the complete FUC rate series with negative-value episodes shaded; the HP-filter robustness check confirms this finding is not an artefact of the smoothing choice). These are not brief technical aberrations: they represent multi-year periods during which any UC-based CPI series would be uninterpretable as a cost-of-living measure.

Resolution: the opportunity cost approach

Three approaches to the negative value problem have been proposed: (i) smoothing π^e via HP filter or rolling windows (reduces incidence, as documented above, but does not eliminate it under extreme appreciation); (ii) replacing time-varying π^e with a long-run constant calibrated to secular fundamentals (removes cyclical sensitivity but makes the index unresponsive to the market conditions that matter most for policy); and (iii) the opportunity cost approach of Diewert (2008)[7]. The OC approach takes $\mathrm{OC}_t^j = \max\{u_t^{f,j} \cdot P_t^H,\ R_t^N\}$: because the new-contract rent R_t^N is strictly positive in every period, the OC index is bounded below by zero by construction (the theoretical justification is in Appendix B, Section B.5; the full regime-classification results are in Appendix G, Section G.5).

Table 5 shows that the OC approach eliminates negative values *entirely* across the full 480-

month Tokyo sample—for all three household types, including during the most extreme phases of the Bubble. This is the first large-scale empirical confirmation that the OC approach functions as Diewert (2008)[7] intended. The mechanism is clean: during investment-regime months when FUC $\leq R^N$ (the dwelling is financially advantageous to hold), the OC index equals R^N—the welfare-layer rent that is always positive. During capital-cost-regime months when FUC $> R^N$, the OC index equals the FUC, which in such periods is already positive. The OC approach thus automatically selects the positive component of the pair $\{\text{FUC} \cdot P^H,\ R^N\}$ in every period, without requiring ad hoc smoothing or truncation.

This finding has direct implications for the European HICP debate. The negative-value problem has been among the two primary technical objections to user cost–based OOH measurement in official statistics (the other being the procurement of quality-adjusted residential property price indices). With the first objection now empirically resolved, the implementation barrier reduces to the second—a data infrastructure question that is addressable through investment in residential property price index programmes, several of which already exist across EU member states. The calibration parameters for all FUC and OC calculations are summarised in Appendix E, Table E.6.

5.2 Rental Equivalence and COLI Consistency

The theoretical ordering derived in Appendix B (Theorem B.1) predicts that the COLI-consistent price of housing services is the new-contract rent R_t^N, which is systematically lower than the unadjusted market rent (by the landlord friction wedge) and lower than the roll-over rent index during appreciation episodes (by the stickiness wedge σ_t). Table 6 confirms both predictions in the Tokyo data.

The ordering $R_t^N \leq R_t^{\text{contract}}$ during appreciation and the reversal during deflation are not merely statistical artefacts: they follow mechanically from the time-dependent adjustment structure established in Section 5.1. With an adjustment hazard of $\hat{\Lambda}_0 \approx 0.001$ per month, the incumbent-rent aggregate responds to a change in the new-contract rent target with an implied half-life of approximately $\ln(2)/\hat{\Lambda}_0 \approx 630$ months. The practical implication is that the official CPI rent index embeds market conditions from many years in the past, making it a poor proxy for the current opportunity cost of shelter at any point in time.

The new-contract rent index R_t^N, by contrast, tracks current market conditions by construction. Over the full sample its mean level (2020=100) is 92.8, compared with 101.2 for the official CPI OOH component—a persistent gap of approximately −8.4 points that reflects the accumulated overhang from the Deflation episode. During the Recent episode (2021–25), the new-contract index has risen above the official index ($\sigma_t > 0$, mean +4.0 points), providing early warning that the CPI is beginning to understate the true cost of shelter in the current cycle—precisely the configuration that matters for inflation-targeting monetary policy.

These results provide empirical support for the recommendation of the CPI Manual (IMF et al., 2025[19]) that the rental equivalence approach be implemented using new-contract rents. In Japan, where the micro-evidence of Shimizu et al. (2010) [15] documents a Calvo parameter of 0.975, the difference between the two rent concepts is quantitatively large enough to shift measured inflation by 1–2 percentage points per year at cyclical peaks—a magnitude sufficient to materially distort the signals available to the Bank of Japan under an inflation-targeting framework.

5.3 The Opportunity Cost Approach and the Welfare–Investment Layer Divergence

The opportunity cost approach, formalised in Section 3.4, takes the maximum of the financial user cost and the new-contract rent as the COLI-consistent period price for OOH: $\mathrm{OC}_t^j = \max\{U_t^{f,j},\ R_t^N\}$. The regime in which $U_t^{f,j} \leq R_t^N$—the *investment regime*—arises when expected capital gains compress the user cost below the prevailing rent; the regime in which $U_t^{f,j} > R_t^N$—the *capital-cost regime*—arises when high interest rates or falling prices raise the cost of ownership above the rental alternative.

Table G.14 (Appendix G, Section G.5) reports the share of months classified into each regime for the baseline FUC Type B ($\lambda = 0.50$), and Figure 4 shows the regime classification together with the OC index and the price-to-rent ratio.

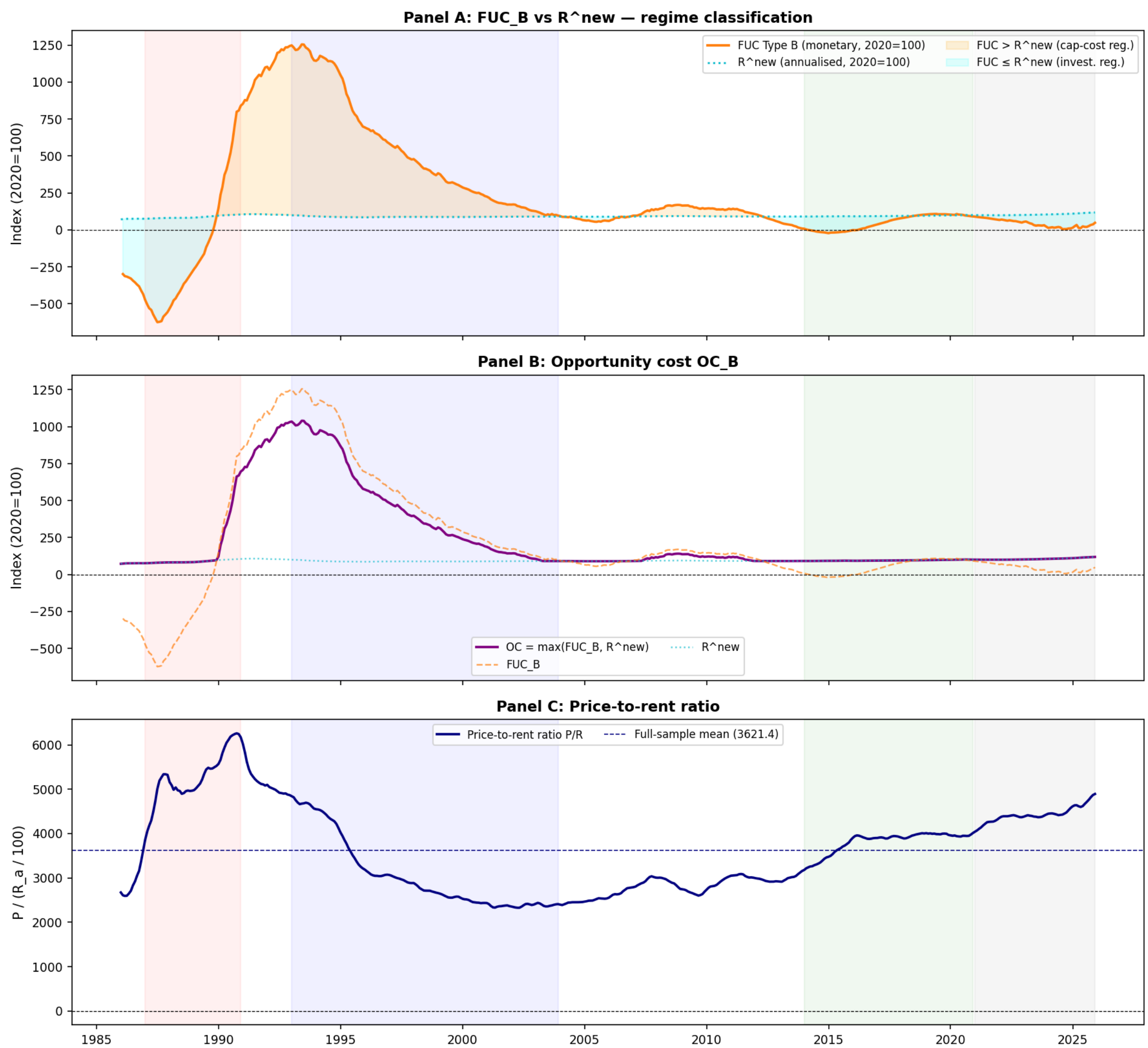

Figure 4: Opportunity cost approach and regime classification (FUC Type B baseline). Panel A: FUC_B vs R^N with investment/capital-cost regime shading. Panel B: OC_B index (2020 = 100). Panel C: Price-to-rent ratio P^H/R^N.

The regime classification reveals a striking asymmetry across episodes that confirms the theoretical predictions of Section 3.4.

During the **Bubble episode** (1987–90), the market was in the investment regime for 75 per cent of months, and the FUC rate turned negative in 70.8 per cent of months. The mean FUC Type B rate was −4.06 per cent per annum (Table G.15; Appendix G, Section G.6), with the HP-smoothed expected appreciation of 14.88 per cent more than offsetting the financing cost of 6.17 per cent (mean prime) plus the carrying cost $c = 0.034$. This is precisely the configuration described by the bubble condition (3.17): expected appreciation exceeded $r_t^f(\lambda) + c$, making the dwelling "pay" the owner to hold it in net financial terms. The acquisitions index registered annual appreciation of 21.6 per cent over the same period—signalling severe inflation at the exact moment when the COLI-consistent user cost was at its *lowest historical level*.

During the **Deflation episode** (1993–03), the regime reversed almost completely: 93.9 per cent of months were classified as capital-cost regime, with the mean FUC Type B rate reaching 14.57 per cent per annum—more than five times its full-sample average. The prime rate fell to 2.78 per cent on average, but the HP-smoothed expected appreciation of −7.14 per cent added a substantial penalty to the user cost, producing the highest ownership costs in the entire 40-year sample. The official CPI OOH index, anchored by sticky roll-over rents (stickiness wedge σ_t averaging −17.4 points), moved in the *opposite direction*—registering the highest level of the series at a mean of 106.4 (2020=100)—while the market was actually imposing exceptional shelter costs on new entrants.

During the **Low-rate episode** (2014–20), 100 per cent of months were in the investment regime, with the mean FUC Type B rate of 1.07 per cent per annum reflecting the near-zero prime rate (mean 1.03 per cent) and positive HP-smoothed appreciation of 4.61 per cent. Despite this, FUC turned negative in 28.6 per cent of months, indicating that even at the long-smoothed appreciation rate, the dwelling's financial benefit periodically exceeded all carrying costs.

During the **Recent episode** (2021–25), 100 per cent of months were in the investment regime. The mean FUC Type B rate of 0.82 per cent per annum—close to the rent-to-price ratio of 2.26 per cent—indicates a market near equilibrium, but with the balance tilted toward the investment regime by the 5.34 per cent HP-smoothed appreciation. The stickiness wedge turned positive (σ_t mean +4.0 points), providing the first extended period in the sample during which the official CPI OOH has understated the current market opportunity cost of shelter.

The sign reversal in equation (3.19)—that $\partial P^H/\partial \pi^e > 0$ while $\partial u^f/\partial \pi^e = -1$—is directly observable in Table G.15: comparing the Bubble and Deflation episodes, the acquisition price rose 21.6 per cent per annum on average during the bubble while the FUC rate was −4.06 per cent, and fell 7.35 per cent during deflation while the FUC rate was 14.57 per cent. The two measures moved in *opposite directions* in both episodes, confirming the welfare–investment layer divergence as a structural feature of the Tokyo housing market over the full 40-year sample.

5.4 CPI Expenditure Weight Bias and Depreciation Sensitivity

Section 3.1 showed theoretically that the acquisitions approach assigns a CPI expenditure weight equal to the value of *net new purchases* of dwellings, which is a small fraction of the total value of housing services flowing from the existing stock. The ratio of the COLI-consistent user cost value to the acquisitions value is given by equation (3.5):

$$\frac{V_U^0}{V_A^0} = \frac{(1+g)(r^*+\delta)}{g+\delta} \tag{5.2}$$

Calibrating this formula with the full-sample Tokyo estimates ($g = 2.87$ per cent, $r^* = 2.69$ per cent, $\delta = 0.020$) yields $V_U^0/V_A^0 \approx 0.99$—approximately unity, in contrast to the baseline calibration value of 1.68 used in Table 1. This result reflects a structural feature of the Tokyo data: the long-run nominal appreciation rate of housing ($g = 2.87$ per cent) closely tracks the average prime rate ($r^* = 2.69$ per cent), making the denominator ($g + \delta = 0.049$) and the numerator's key term ($r^* + \delta = 0.047$) nearly equal. In the theoretical baseline, $r^* = 0.030 > g = 0.010$, so the ratio substantially exceeds unity; in the Tokyo data, the secular trend of rising asset prices over four decades has compressed this gap.

Sensitivity analysis confirms the linearity of the user cost rate with respect to the depreciation rate: $\partial u_t / \partial \delta = 1$ in every period, so a change of ± 0.002 (i.e., ± 0.2 percentage points) in δ shifts the FUC rate by exactly ± 0.002 per annum in every observation. Table G.16 reports the corresponding changes in the V_U^0 / V_A^0 ratio. The robustness of this calibration is grounded in the hedonic evidence: Appendix C (Section C.3) presents two independent methods for estimating δ from Tokyo condominium transaction data, and Table C.7 shows that both the pooled estimate ($\hat{\delta} = 0.0204$, SE $= 0.0003$, $N = 357{,}627$) and the rolling-window mean ($\hat{\delta} = 0.0203$ across 467 windows) lie within one basis point of the MLIT benchmark of $\delta = 0.020$. This convergence across estimation methods—Method 1 (the nonlinear builder's model) and Method 2 (the log-linear age coefficient)—implies that the user cost sensitivity to δ documented in Table G.16 is an upper bound on the true calibration uncertainty: since the two hedonic methods agree to within ± 0.001, the effective band is ± 0.001 rather than ± 0.002, and the implied change in V_U^0 / V_A^0 is less than 0.001—negligible in economic terms.

The near-unity weight ratio has an important implication for CPI construction in Japan. The standard argument against the acquisitions approach—that it assigns an expenditure weight far below the true welfare-consistent value—applies most strongly in economies where the real interest rate substantially exceeds the long-run appreciation rate. In Japan, the combination of prolonged low interest rates and secular asset price appreciation has compressed this differential, making the weight distortion from the acquisitions approach relatively small in magnitude compared with the *price* distortion documented in the previous subsection. The dominant source of CPI bias in the Tokyo data is therefore not the weight channel but the price channel: the systematic procyclicality of the acquisitions index and the persistent undershoot of the sticky-rent CPI relative to the new-contract benchmark.

5.5 Implications for Monetary Policy and CPI Reform

The empirical findings of Sections 5.1–5.4 converge on a conclusion with direct implications for three audiences: central banks operating under inflation-targeting frameworks, national statistical offices seeking to incorporate OOH into their CPIs, and the European institutions that have been unable to resolve the HICP impasse since 2014.

Cyclical mismeasurement and monetary policy. The most striking finding is the *directional* pattern of divergence between the official CPI OOH component and the COLI-consistent alternatives. Table 6 reports the mean divergence of each alternative from the official CPI OOH in index points (2020 = 100) by episode.

Table 6: Divergence from Official CPI OOH by Measurement Approach and Market Episode (index points, 2020 = 100)

Approach	Bubble	Deflation	Low-rate	Recent	Full
Acquisitions − CPI	+27.2	−37.3	−8.3	+16.8	−15.3
Basic UC (HP-smoothed) − CPI	−622.4	+1113.9	−111.2	−166.2	+314.3
Payments B ($\lambda = 0.50$) − CPI	+174.1	−4.3	−8.0	+31.5	+24.1
FUC Type A ($\lambda = 0.20$) − CPI	−188.2	+344.4	−42.4	−43.9	+94.0
FUC Type B ($\lambda = 0.50$) − CPI	−234.5	+426.5	−49.8	−56.9	+117.5
FUC Type C ($\lambda = 0.80$) − CPI	−306.1	+553.5	−61.1	−77.1	+153.8
R^N (new-contract) − CPI	−3.1	−17.4	−5.1	+4.0	−8.4

Note: All series normalised to 2020 = 100 before differencing. Positive values indicate the alternative is *above* the official CPI OOH; negative values indicate it is *below*. FUC uses the monetary index $u_t^j \times P_t^H$; OC values (not shown) equal OC $\times P_t^H/\bar{P}_{2020}^H \times 100$ and are always positive (Table 5).

Three findings are particularly relevant for monetary policy.

First, the FUC approaches diverge from the official CPI in a *counter-cyclical* direction: substantially below the CPI during the Bubble (FUC Type B: −234.5 points) and substantially above during the Deflation (+426.5 points), a full-cycle swing of 661 index points. Under a standard Taylor rule with a response coefficient of 1.5, the Bubble-era understatement of shelter costs would have biased the policy rate downward by approximately 2–3 percentage points, reinforcing the credit expansion that sustained the bubble. The Deflation-era overstatement would have similarly biased the policy rate upward, deepening and prolonging the contraction.

Second, the payments approach exhibits the *opposite* procyclicality to the FUC: above the official CPI during the Bubble (+174.1 points, reflecting high nominal mortgage rates) and approximately aligned during the Deflation (−4.3 points). This pattern is driven not by changes in the true welfare cost of shelter but by the nominal interest rate cycle, confirming the theoretical critique of Lebow and Rudd (2003)[40] and the caution of the CPI Manual (IMF et al., 2025[19]) against its use as a primary OOH measure.

Third, the new-contract rent index R_t^N—and, closely tracking it, the OC approach—provides the most stable and policy-relevant alternative. Its divergence from the official CPI is modest in absolute magnitude, ranging from −17.4 (Deflation) to +4.0 (Recent), and switches sign appropriately across regimes. During the Recent episode (2021–25), the stickiness wedge has turned positive (mean +4.0 points, maximum +15.7 points), indicating that the official CPI OOH is currently *understating* the market opportunity cost of shelter. For the Bank of Japan, which began rate normalisation in 2024, this understatement implies that the true cost of living faced by households seeking shelter is rising faster than the official CPI records.

The price channel dominates the weight channel. The finding in Section 5.4 that $V_U^0/V_A^0 \approx 0.99$ in the Tokyo data—compared with the theoretical calibration value of 1.68—has an important practical implication. The dominant source of OOH measurement bias in the Tokyo data is the *price channel*: the procyclical acquisitions index and the lagging sticky-rent official CPI. The *weight channel*, which has received most attention in the theoretical literature (Diewert and Nakamura, 2011[9]), is of secondary importance when the long-run asset appreciation rate closely tracks the average financing cost—a configuration that has become common across major OECD cities since the mid-2010s. This finding shifts the priority order for measurement reform: correcting the price measure (via new-contract rents or the OC approach) yields larger welfare gains than recalibrating expenditure weights.

Bridging to the reform roadmap. The three pillars of the empirical analysis—the empirical resolution of the negative user cost problem via the OC approach (Section 5.1), the quantification of the stickiness wedge between new-contract and incumbent rents (Section 5.2), and the dominance of the price channel over the weight channel in Japan (Section 5.4)—together justify a concrete three-step reform roadmap. Step 1 (immediately feasible for any statistical agency with an existing rent survey) is the transition to new-contract rents (Appendix A, Section A.2; Appendix F, Section F.6). Step 2 (feasible for agencies with established residential property price indices, including several EU member states) is the supplementary publication of the OC index, which this paper has shown to be free from negative values in practice. Step 3 (longer term) is the introduction of leverage-differentiated FUC sub-indices to make visible the distributional dimension of OOH costs. This roadmap is developed in detail in Section 6.

A counterfactual: Taylor-rule policy rates under alternative OOH measurement. To illustrate the policy stakes of OOH measurement concretely, Figure 5 constructs a counterfactual Taylor rule exercise. We replace the official CPI OOH component with the new-contract rent-based alternative R_t^N—the most immediately implementable reform—and recompute the standard Taylor rule (equation 5.3):

$$i_t^* \; = \; r^{\text{neu}} + \pi^* + \phi\big(\pi_t^{\text{CPI}} - \pi^*\big), \quad r^{\text{neu}} = \pi^* = 2\%, \;\; \phi = 1.5 \tag{5.3}$$

Panel A of Figure 5 shows the two CPI inflation series, and Panel B shows the corresponding Taylor-implied policy rates together with the actual BOJ prime rate.

The counterfactual reveals a systematic and cyclically asymmetric policy bias. During the **Bubble episode** (1987–90), the R^N-based CPI registers average inflation of 3.1 per cent per annum— 1.3 percentage points above the official CPI (1.8 per cent). Under the Taylor rule with $\phi = 1.5$, this difference implies a *tighter* counterfactual policy rate averaging +2.0 percentage points above the official-CPI path (maximum +4.6 pp in 1990:Q1). The actual BOJ prime rate averaged 6.2 per cent, but the Taylor rule applied to the official CPI implies only 3.7 per cent: the official CPI was signalling that policy was already tight, when in reality the new-contract rent index was recording a sustained inflationary premium. Had the Bank applied the Taylor rule to the R^N-based CPI, the implied rate of 5.7 per cent would have been substantially closer to the actual rate, providing a stronger theoretical rationale for the 1989–90 tightening cycle.

During the **Deflation episode** (1993–2003), the asymmetry reverses. The R^N-based CPI registers average deflation of 0.3 per cent per annum, while the official CPI is close to zero (−0.02 per cent). The counterfactual implied rate averages −0.5 pp *below* the official-CPI path, reaching a maximum gap of −4.4 pp at the trough of the deflation. Under the R^N metric, the Taylor rule signals a need for *more aggressive easing* than the official CPI would suggest—consistent with the observed persistence of the Lost Decades deflation and the subsequent literature attributing the Bank of Japan's policy stance to an excessively benign reading of the official shelter cost component (Hoshi and Kashyap, 2004[26]).

During the **Recent episode** (2021–25), the R^N-based CPI runs modestly above the official CPI (mean gap +0.7 pp), implying a counterfactual Taylor rate averaging +0.7 pp higher than the official-CPI path. This is consistent with the stickiness wedge turning positive (mean +4.0 index points) during this period and provides empirical support for the view that the Bank of Japan's normalisation cycle, begun in 2024, is somewhat behind the curve relative to what a R^N-based CPI would imply.

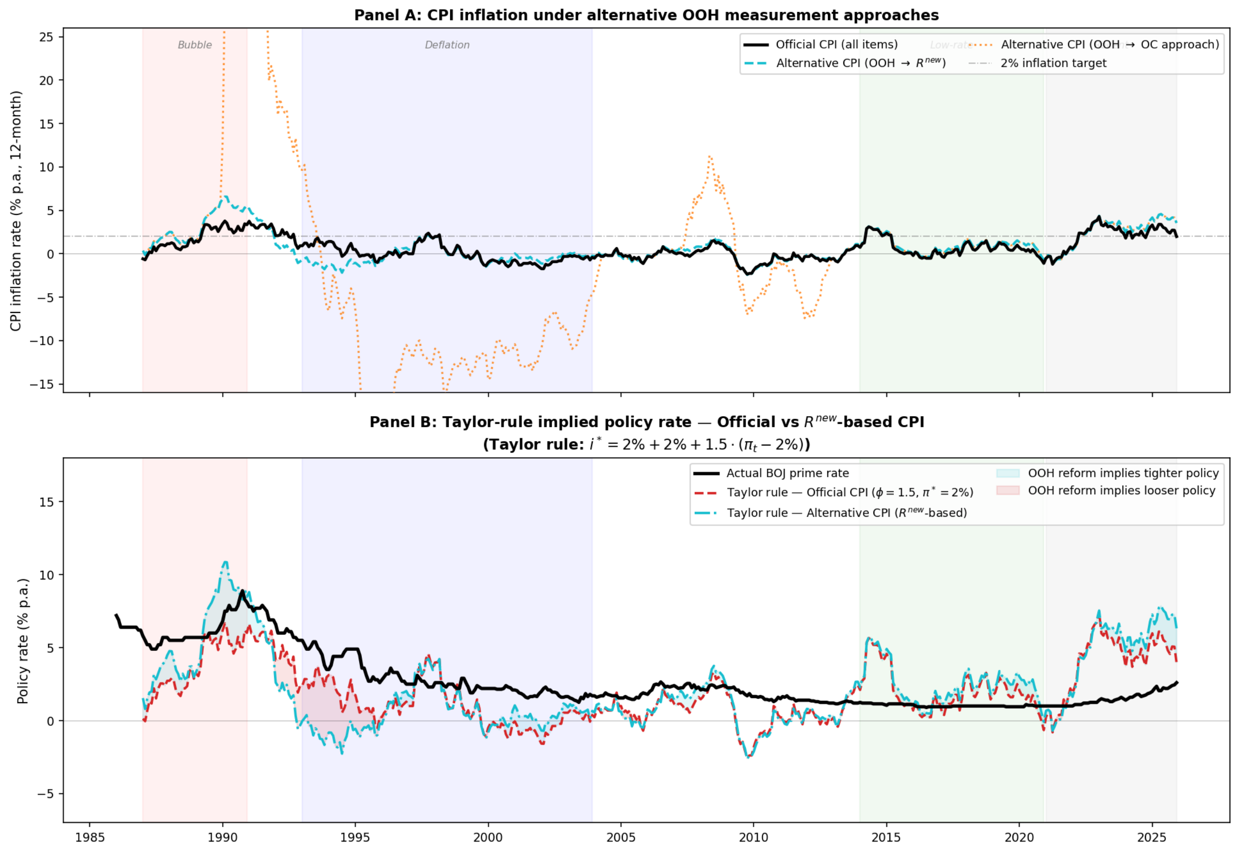

Figure 5: Taylor-rule counterfactual: policy implications of replacing the official CPI OOH component with the new-contract rent index R^N. *Panel A:* CPI inflation rate (% p.a., 12-month log change) under the official CPI, the R^N-based alternative, and the OC-based alternative. *Panel B:* Taylor-rule implied policy rate ($i^* = 2\% + 2\% + 1.5(\pi_t - 2\%)$) under the official and R^N-based CPIs, together with the actual BOJ prime rate. Shaded regions: Bubble (red), Deflation (blue), Low-rate (green), Recent (grey).

6 Conclusion

This paper has developed a unified theoretical framework that nests five approaches to the measurement of owner-occupied housing (OOH) services—acquisitions, rental equivalence, user cost, payments, and opportunity cost—as special cases of the single identity $u_t = r_t^\dagger + c - \pi_t^\dagger$, where the pair $(r_t^\dagger, \pi_t^\dagger)$ characterises each approach's treatment of the opportunity cost of capital and of expected capital gains. We have implemented all five approaches on a common dataset of approximately 3.1 million property transactions and rental contracts for the Tokyo Special District (23 wards) over the 40-year period 1986–2025, allowing a direct comparison free from the compositional and data-source differences that confound most existing comparative studies. The central empirical finding is that the choice of measurement approach is not a matter of minor statistical convention: during the late-1980s Bubble episode, the acquisitions index registered annual appreciation of 21.6 per cent while the Financial User Cost (FUC) type B rate stood at −4.06 per cent; during the Deflation of 1993–2003, the acquisitions index fell at 7.4 per cent per annum while the FUC rate reached 14.6 per cent. The two dominant approaches moved in *opposite directions* in both episodes, confirming the theoretical prediction that the investment layer and the welfare layer of housing diverge systematically across the asset-price cycle.

Tokyo as a laboratory for extreme cases and universal lessons. Tokyo occupies an unusual position in the international literature on OOH measurement: it is simultaneously one of the most extreme cases and one of the best-documented. The Calvo rent-adjustment parameter estimated from the full 40-year panel is $1 - \hat{\Lambda}_0 = 0.9989$—approximately 34 times higher than the corresponding estimate of 0.71 for the United States (Genesove, 2003[10])—implying that the official CPI rent index embeds market conditions from several decades in the past. The full-sample stickiness wedge averages −8.4 index points (2020 = 100), and swings from −17.4 during the Deflation to +4.0 during the Recent episode, indicating that the official CPI has variously overstated and understated the true opportunity cost of shelter by economically significant margins.

Precisely because these features are extreme, Tokyo provides the most stringent test yet conducted of the five OOH measurement approaches. The lessons are transferable. First, the structural condition that generates large measurement divergences—expected appreciation π^e that substantially exceeds or falls short of the financing cost $r^f(\lambda) + c$— is not unique to Japan. The same condition was present in the United States and the United Kingdom during the 2000s bubble, in Australia during the 2010s, and in most major OECD cities during the post-pandemic appreciation of 2021–25. Second, the finding that $V_U^0/V_A^0 \approx 1$ in the Tokyo data—in contrast to the theoretical calibration value of 1.68 (Appendix B, Section B.4)—reflects the compressed differential between the long-run asset appreciation rate ($g = 2.87$ per cent) and the average prime rate ($r^* = 2.69$ per cent) over four decades (depreciation sensitivity in Appendix G, Section G.7). This configuration, in which prolonged low interest rates and secular asset price appreciation are simultaneous, is shared by many cities that have experienced sustained house-price booms since the mid-2010s. In this environment, the dominant source of CPI bias is the *price* channel—the procyclical acquisitions index and the lagging sticky-rent official CPI—rather than the *weight* channel, a finding that reshapes the priority order for measurement reform.

The negative user cost problem: diagnosis and resolution. The most persistent technical obstacle to the adoption of user cost approaches in official statistics is the possibility of negative user cost values during episodes of rapid house-price appreciation. The Tokyo data quantify this problem at an unprecedented scale. The Basic User Cost computed with realised appreciation ($u_t^{\text{basic,act}} = r_t^{\text{prime}} + c - \pi_t^{\text{act}}$) is negative in 36.2 per cent of the 480-month sample—174 months in total—with a minimum value of -73.2 per cent per annum during the height of the Bubble. Even with HP-filter smoothing, the Basic UC remains negative in 29.4 per cent of months and in 72.9 per cent of Bubble-episode months. The FUC reduces the incidence of negative values by raising the threshold through the financing premium—for Type B with a Bubble-era prime rate averaging 6.2 per cent, the zero-crossing threshold is $\pi^{e,*} \approx 10.8$ per cent, which was nonetheless exceeded by the HP-smoothed appreciation of 14.9 per cent for 34 consecutive months (January 1987 to October 1989). This magnitude of persistent negative user costs renders any UC-based CPI series uninterpretable as a cost-of-living measure for sustained periods.

The empirical analysis demonstrates, however, that the opportunity cost (OC) approach of

Diewert (2008)[7]—taking $\text{OC}_t = \max\{\text{FUC}_t \cdot P_t^H,\ R_t^N\}$—resolves this problem completely and without further smoothing. Because the new-contract rent R_t^N is strictly positive in all periods, the OC index is bounded below by zero by construction. In the Tokyo data, all three OC variants (Types A, B, and C) record *zero months* with negative values across the full 480-month sample—a finding that holds even during the most extreme phases of the Bubble. The OC index automatically selects the welfare-layer rent during investment-dominated regimes (when $\text{FUC} \leq R^N$) and the capital-cost-layer FUC during capital-cost regimes (when $\text{FUC} > R^N$), producing a series that is always non-negative, always interpretable, and always responsive to current market conditions.

Three approaches to resolving negative UC values have been proposed in the literature: (i) HP-filter or rolling-window smoothing of expected appreciation; (ii) use of a long-run constant appreciation rate calibrated to secular fundamentals; and (iii) adoption of the OC approach. The Tokyo evidence indicates that approaches (i) and (ii) reduce but do not eliminate the problem when appreciation is exceptionally large and persistent, while approach (iii) eliminates it entirely. We therefore concur with Diewert (2008)[7] and Diewert and Nakamura (2011)[9] that the OC approach is the preferred implementation of the user cost principle for official statistics, and we provide the first large-scale empirical confirmation that it functions as intended.

Implications for national statistical offices. Eurostat has sought since 2014 to incorporate OOH into the Harmonised Index of Consumer Prices (HICP) but has been unable to reach methodological agreement among EU member states. As of 2025, the HICP continues to exclude OOH, and the experimental OOH-PI published by Eurostat uses the acquisitions approach—which this paper identifies as the approach most prone to procyclical mismeasurement. The deadlock reflects two related concerns: the absence of theoretical consensus on which approach is COLI-consistent, and the practical difficulty of implementing user cost methods that can produce negative values.

The present paper addresses both concerns directly. On the first, the theoretical ordering of Appendix B (Theorem B.1) establishes that the adjusted new-contract rent and the FUC are the only two COLI-consistent measures under competitive equilibrium, and that the OC approach provides an upper bound that is always non-negative. On the second, the Tokyo evidence demonstrates that the OC approach eliminates negative values in practice, even under extreme bubble conditions (leverage-type incidence in Appendix G, Section G.3; full regime-classification in Section G.5). We propose a three-step implementation roadmap for national statistical offices.

Step 1 (immediately feasible): Transition from incumbent (roll-over) rents to new-contract rents in the rental equivalence approach (methodology in Appendix A, Section A.2; Tokyo empirical contrast in Appendix F, Section F.6). This requires no new data collection beyond a reorientation of existing rent surveys toward newly signed leases. In Japan, this transition would eliminate a stickiness wedge that averages -8.4 index points over the full sample and exceeds -17 points during the Deflation. For countries with lower rent stickiness than Japan—including Germany (Calvo parameter ≈ 0.78) and the United States (≈ 0.29)—the gain is smaller in magnitude but still economically significant at cyclical peaks (Hoffmann and Kurz, 2002[18]; Genesove, 2003[10]). The 2025 edition of the CPI Manual (IMF et al., 2025[19]) already recommends this transition; this paper provides the quantitative evidence to motivate its priority.

Step 2 (medium term): Publish the OC index as a supplementary series alongside the official CPI. Countries with established residential property price indices (RPPIs)—including the Netherlands, France, Germany, and the Nordic countries—can implement this step without significant new infrastructure. The negative-value problem that has made user cost approaches appear impractical in official statistics is resolved by the OC formulation, as documented above. Iceland's existing implementation of a user cost–based CPI (Gudnason and Jonsdottir, 2011[33]) provides the institutional precedent; the OC approach is the natural extension that removes the remaining technical concern about negative values. Fenwick (2009, 2012)[4][5] argued that statistical agencies should publish a range of OOH indices suited to different analytical purposes; the OC index would be the welfare-consistent member of such a suite.

Step 3 (longer term): Introduce FUC variants differentiated by household leverage type, drawing on administrative data on the distribution of loan-to-value ratios across the housing stock. The constant financing-structure spread of 0.90 percentage points between the low-leverage (Type A) and high-leverage (Type C) FUC variants documented in this paper means that even a simple two-type decomposition would capture the bulk of the distributional heterogeneity in OOH costs. Publishing leverage-differentiated FUC indices would make visible the generational divide between existing homeowners (whose costs are anchored to historical purchase prices) and new market entrants (who face current asset prices and current financing rates)—a dimension of inequality that is invisible in any aggregate index.

Implications for monetary policy. The CPI measurement biases documented in this paper are not merely statistical artefacts: they translate directly into policy miscalibration under inflation-targeting frameworks. The FUC Type B index diverges from the official CPI OOH component by −234.5 index points during the Bubble and +426.5 points during the Deflation—a full-cycle swing of 661 index points. Under a standard Taylor rule with a response coefficient of 1.5, the Bubble-era understatement of shelter costs would have biased the policy rate downward by approximately 2–3 percentage points, reinforcing the credit expansion that sustained the bubble. The Deflation-era overstatement would have similarly biased the policy rate upward, deepening and prolonging the contraction.

The Recent episode (2021–25) carries immediate policy relevance. The stickiness wedge has turned positive (mean +4.0 index points, maximum +15.7 points), indicating that the official CPI OOH component is currently *understating* the new-contract rent benchmark.

For the Bank of Japan, which has operated in an inflation- targeting framework since the 2010s and has begun to normalise rates from 2024, this understatement implies that the true cost of living faced by households seeking shelter in the current market is rising faster than the official CPI records. More broadly, the post-pandemic appreciation observed across major global cities—Tokyo, London, Sydney, Vancouver, New York—suggests that the measurement problem identified in this paper is a live issue for central banks worldwide, not a historical curiosity confined to Japan's bubble era.

Limitations and directions for future research. The empirical analysis of this paper is conducted at the aggregate level, treating the OOH stock as if all households faced the same quality-adjusted price and rent indices. Diewert and Nakamura (2011)[9] demonstrated formally that a fully welfare-consistent treatment requires *heterogeneous-agent* sub-indices differentiated by tenure status, entry cohort, and financing structure. Constructing such sub-indices requires household-level data on purchase prices, outstanding mortgage balances, and renovation expenditures that are rarely available at the frequency and coverage required for monthly CPI compilation. Developing administrative data linkages between property transaction records and household survey data is therefore a priority for future work.

Second, the Tokyo findings should be replicated in other major metropolitan areas to establish which features of the results are specific to Japan's institutional environment— in particular, the exceptional rent stickiness under the *Shakuya Hō̄*—and which are shared by cities that have experienced comparable asset-price cycles. The simultaneous availability of quality-adjusted price and rent indices from the same sampling frame, which is the methodological foundation of the present study, is a prerequisite for such comparisons and highlights the value of investing in integrated property information platforms.

Third, the analysis has focused on condominium properties in the Tokyo Special District (23 wards). The extension to single-family houses, where the land–structure decomposition is more complex (Appendix D), and to smaller cities and rural areas, where thin rental markets make the rental equivalence approach less reliable, raises additional methodological challenges that merit separate investigation.

The central conclusion of this paper is simple to state even if its quantitative implementation is demanding: no single OOH measurement approach is adequate for all purposes and all market regimes. The unified framework $u_t = r_t^\dagger + c - \pi_t^\dagger$ clarifies what each approach measures and for what purpose it is suited. The empirical evidence from four decades of Tokyo data shows that the consequences of choosing the wrong approach—or of failing to choose at all, as in the European HICP—are measured in hundreds of index points of CPI divergence, in percentage points of policy rate miscalibration, and in the systematic misrepresentation of the welfare costs of the largest component of household consumption. The OC approach, grounded in the Könus COLI and free from negative-value problems, together with a transition to new-contract rents, offers the most practical and theoretically consistent path forward for statistical agencies seeking to resolve this long-standing measurement impasse.

References

[1] Crone, T.M., L.I. Nakamura and R. Voith (2000), "Measuring Housing Services Inflation", *Journal of Economic and Social Measurement* 26, 153–171.

[2] Eurostat, IMF, OECD, UN and World Bank (1993), *System of National Accounts (1993)*, United Nations: New York.

[3] Eurostat (2017), *Technical Manual on Owner-Occupied Housing and House Price Indices*, Brussels: European Commission.

[4] Fenwick, D. (2009), "A Statistical System for Residential Property Price Indices", Eurostat-IAOS-IFC Conference on Residential Property Price Indices, Bank for International Settlements, November.

[5] Fenwick, D. (2012), "A Family of Price Indices", The Meeting of Groups of Experts on Consumer Price Indices, UNECE/ILO, United Nations Palais des Nations, Geneva Switzerland, May 30-June 1.

[6] Diewert, W.E. (2005a), "Issues in the Measurement of Capital Services, Depreciation, Asset Price Changes and Interest Rates", pp. 479–542 in *Measuring Capital in the New Economy*, C. Corrado, J. Haltiwanger and D. Sichel (eds.), Chicago: University of Chicago Press.

[7] Diewert, W.E. (2008), "OECD Workshop on Productivity Analysis and Measurement: Conclusions and Future Directions", pp. 11–36 in *Productivity Measurement and Analysis*, Berne: OECD and Swiss Federal Statistical Office.

[8] Diewert, W.E., A.O. Nakamura and L.I. Nakamura (2009), "The Housing Bubble and a New Approach to Accounting for Housing in a CPI", *Journal of Housing Economics* 18(3), 156–171.

[9] Diewert, W.E. and A.O. Nakamura (2011), “Accounting for Housing in a CPI”, pp. 7–32 in Diewert, W.E., B.M. Balk, D. Fixler, K.J. Fox and A.O. Nakamura, *Price and Productivity Measurement: Volume 1: Housing*, Victoria: Trafford Press.

[10] Genesove, D. (2003), “The Nominal Rigidity of Apartment Rents”, *The Review of Economics and Statistics* 85(4), 844–853.

[11] Goodhart, C. (2001), “What Weights should be Given to Asset Prices in the Measurement of Inflation?”, *The Economic Journal* 111(June), F335–F356.

[12] Hulten, C.R. (1990), “The Measurement of Capital”, pp. 119–158 in *Fifty Years of Economic Measurement*, E. R. Berndt and J. E. Triplett (eds.), Chicago: the University of Chicago Press.

[13] ILO, Eurostat, IMF, OECD, UNECE and the World Bank (2004), *Consumer Price Index Manual: Theory and Practice*, Peter Hill (ed.), Geneva: International Labour Office.

[14] Shimizu, C., W.E. Diewert, K. Nishimura and T. Watanabe (2012), “The Estimation of owner-occupied Housing Indexes using the RPPI: The Case of Tokyo”, Meeting of the Group of Experts on Consumer Price Indices, Geneva, May 28.

[15] Shimizu, C., K.G. Nishimura and T. Watanabe (2010), “Residential Rents and Price Rigidity: Micro Structure and Macro Consequences”, *Journal of Japanese and International Economies* 24, 282–299.

[16] Shimizu, C. and T. Watanabe (2011), “Nominal Rigidity of Housing Rent”, *Financial Review* 106(1), 52–68.

[17] Shimizu, C., K.G. Nishimura and T. Watanabe (2016), “House Prices at Different Stages of the Buying/Selling Process”, *Regional Science and Urban Economics* 59, 37–53. `http://dx.doi.org/10.1016/j.regsciurbeco.2016.04.001`

[18] Hoffmann, J. and C. Kurz (2002), "Rent Indices for Housing in West Germany: 1985 to 1998", Discussion Paper 01/02, Economic Research Centre of the Deutsche Bundesbank, Frankfurt.

[19] IMF, ILO, Eurostat, OECD, UNECE and the World Bank (2025), *Consumer Price Index Manual: Theory and Practice*, 2025 Edition, Washington D.C.: International Monetary Fund.

[20] Jorgenson, D.W. (1963), "Capital Theory and Investment Behavior", *American Economic Review* 53(2), 247–259.

[21] Calvo, G.A. (1983), "Staggered Prices in a Utility-Maximizing Framework", *Journal of Monetary Economics* 12(3), 383–398.

[22] Poterba, J.M. (1984), "Tax Subsidies to Owner-Occupied Housing: An Asset-Market Approach", *Quarterly Journal of Economics* 99(4), 729–752.

[23] Campbell, S.D., M.A. Davis, J. Gallin and R.F. Martin (2009), "What Moves Housing Markets: A Variance Decomposition of the Rent-Price Ratio", *Journal of Urban Economics* 66(2), 90–102.

[24] Himmelberg, C., C. Mayer and T. Sinai (2005), "Assessing High House Prices: Bubbles, Fundamentals and Misperceptions", *Journal of Economic Perspectives* 19(4), 67–92.

[25] Shimizu, C. and K.G. Nishimura (2007), "Pricing Structure in Tokyo Metropolitan Land Markets and its Structural Changes: Pre-bubble, Bubble, and Post-bubble Periods", *Journal of Real Estate Finance and Economics* 35(4), 475–496.

[26] Hoshi, T. and A.K. Kashyap (2004), "Japan's Financial Crisis and Economic Stagnation", *Journal of Economic Perspectives* 18(1), 3–26.

[27] Bureau of Labor Statistics (1983), *Trends in Multifactor Productivity, 1948–81*, Bulletin 2178, Washington D.C.: U.S. Department of Labor.

[28] Crone, T.M., L.I. Nakamura and R. Voith (2011), "Rents Have Been Rising, Not Falling, in the Postwar Period", *The Review of Economics and Statistics* 93(2), 628–642.

[29] Diewert, W.E. (1974), "Intertemporal Consumer Theory and the Demand for Durables", *Econometrica* 42(3), 497–516.

[30] Diewert, W.E. and C. Shimizu (2015), "Residential Property Price Indices for Tokyo", *Macroeconomic Dynamics* 19(8), 1659–1714.

[31] Diewert, W.E. and C. Shimizu (2016), "Hedonic Regression Models for Tokyo Condominium Sales", *Regional Science and Urban Economics* 60, 300–315.

[32] Garner, T.I. and R. Verbrugge (2011), "The Puzzling Divergence of Rents and User Costs, 1980–2004: Summary and Extensions", pp. 125–146 in Diewert, W.E., B.M. Balk, D. Fixler, K.J. Fox and A.O. Nakamura, *Price and Productivity Measurement: Volume 1: Housing*, Victoria: Trafford Press.

[33] Gudnason, R. and G.R. Jonsdottir (2011), "Owner Occupied Housing in the Icelandic CPI", pp. 85–96 in Diewert, W.E., B.M. Balk, D. Fixler, K.J. Fox and A.O. Nakamura, *Price and Productivity Measurement: Volume 1: Housing*, Victoria: Trafford Press.

[34] Hall, R.E. (1967), "Technical Change and Capital from the Point of View of the Dual", *Review of Economic Studies* 35(1), 35–46.

[35] Heston, A. and A.O. Nakamura (2011), "Reported Prices and Rents of Housing: Reflections of costs, income or both?", pp. 117–124 in Diewert, W.E., B.M. Balk, D. Fixler, K.J. Fox and A.O. Nakamura, *Price and Productivity Measurement: Volume 1: Housing*, Victoria: Trafford Press.

[36] Hill, R.J. (2013), "Hedonic Price Indexes for Residential Housing: A Survey, Evaluation and Taxonomy", *Journal of Economic Surveys* 27(5), 879–914.

[37] Hill, R.J., M. Scholz, C. Shimizu and M. Steurer (2018), “Can National Statistical Agencies Measure Consumer Inflation Better by Using Scanner Data? The Case of Supermarkets”, *Review of Income and Wealth* 64(2), 390–417.

[38] Hulten, C.R. and F.C. Wykoff (1981a), “The Estimation of Economic Depreciation using Vintage Asset Prices: An Application of the Box–Cox Power Transformation”, *Journal of Econometrics* 15(3), 367–396.

[39] Hulten, C.R. and F.C. Wykoff (1981b), “The Measurement of Economic Depreciation”, pp. 81–125 in C.R. Hulten (ed.), *Depreciation, Inflation and the Taxation of Income from Capital*, Washington D.C.: The Urban Institute Press.

[40] Lebow, D.E. and J.B. Rudd (2003), “Measurement Error in the Consumer Price Index: Where Do We Stand?”, *Journal of Economic Literature* 41(1), 159–201.

[41] Shimizu, C., K.G. Nishimura and T. Watanabe (2010a), “Residential Rents and Price Rigidity: Micro Structure and Macro Consequences”, *Journal of Japanese and International Economies* 24, 282–299.

[42] Suzuki, M., Y. Asami and C. Shimizu (2021), “Unit-level Price Rigidity in Rental Housing Markets: Evidence from Tokyo”, *Journal of Housing Economics* 52, 101749.

[43] Verbrugge, R. (2008), “The Puzzling Divergence of Rents and User Costs, 1980–2004”, *Review of Income and Wealth* 54(4), 671–699.

[44] Hill, R.J., M. Steurer and S.R. Waltl (2017), “Owner Occupied Housing, CPI and the Rental Equivalence Approach: A Simulation Study”, Discussion Paper SFB, Graz University of Technology.

[45] Muth, R.F. and E. Nourse (1963), “The Relationship Between the User Cost of and Demand for Housing”, pp. 72–78 in *Proceedings of the Business and Economics Statistics Section, American Statistical Association.*

Online Appendix

Measuring the Services of Durables and Owner Occupied Housing: A Unified Framework and Forty Years of Tokyo Evidence

W. Erwin Diewert[†] and Chihiro Shimizu[‡]

March 2026

Overview. This Online Appendix contains theoretical derivations, depreciation models, hedonic regression frameworks, and supplementary empirical results referenced in the main text. The appendices are organized as follows.

- **A**: The three main approaches to OOH measurement: computation, advantages, and disadvantages.
- **B**: Theoretical relationships among the three approaches under the cost-of-living index framework.
- **C**: Depreciation models: general framework, straight line, and one hoss shay.
- **D**: Hedonic regression models for housing: land–structure decomposition and demand-side models.
- **E**: The financial user cost: Type A, Type B, and Type C.
- **F**: Construction of quality-adjusted price and rent indices.
- **G**: Supplementary empirical results: user cost estimation, index comparisons, and robustness analysis.

[†]School of Economics, University of British Columbia, Vancouver B.C., Canada, V6T 1Z1. E-mail: erwin.diewert@ubc.ca.

[‡]School of Social Data Science, Hitotsubashi University, 2-1 Naka, Kunitachi, Tokyo 186-8601, Japan. E-mail: c.shimizu@r.hit-u.ac.jp. Financial support from JSPS KAKENHI Grant (S) 24H00012 is gratefully acknowledged.

A The Three Main Approaches to OOH Measurement: Computation, Advantages, and Disadvantages

This appendix provides a detailed treatment of the three main approaches to measuring owner-occupied housing (OOH) services in the Consumer Price Index (CPI). For each approach, we describe (i) the theoretical motivation, (ii) the step-by-step computation procedure, and (iii) the principal advantages and disadvantages. The fourth approach considered in the main text—the opportunity cost approach—is a synthesis of the rental equivalence and user cost approaches and is described in Section 3.4 of the main paper.

A.1 The Acquisitions Approach

Theoretical Motivation

The acquisitions approach treats the purchase of an owner-occupied dwelling symmetrically with the purchase of any non-durable consumption good: the entire purchase price is attributed to the period of acquisition. The underlying rationale is that measuring *monetary expenditures* on new dwellings—rather than imputed service flows—provides a transactions-based measure of inflation that requires minimal imputation.[4]

Computation

Step 1: Define the scope of transactions. The acquisitions approach covers *net purchases* of dwellings by the household sector from other institutional sectors. In practice, this consists primarily of:

[4] Goodhart (2001, p. F350) describes the first of the main approaches as follows: "The first is the net acquisition approach, which is the change in the price of newly purchased owner-occupied dwellings, weighted by the net purchases of the reference population. This is an asset-based measure, and therefore comes close to my preferred measure of inflation as a change in the value of money, though the change in the price of the stock of existing houses rather than just of net purchases would in some respects be even better." Eurostat (2017) provides a comprehensive description of the approach.

- Purchases of newly constructed dwelling units from builders or developers.
- Net purchases of second-hand dwellings from non-household sectors (e.g., local governments selling public housing).

Purchases of existing dwellings between households cancel out in aggregate and are therefore excluded.

Step 2: Construct the price index. The price index for the acquisitions approach is a *quality-adjusted price index for new dwellings*, P_t^{acq}. Two primary methods are used:

(a) **Construction cost index.** If the land component of new house prices is excluded (as in the Eurostat HICP framework), the price index is approximated by a construction cost index:

$$P_t^{\text{acq}} \approx \frac{C_t(1+\mu_t)}{C_0(1+\mu_0)}, \tag{A.1}$$

where C_t is the construction cost per square metre in period t and μ_t is the builder's profit margin.

(b) **Hedonic price index.** When transaction data on new dwellings are available, a hedonic regression of the form

$$\ln P_{n,t}^H = \alpha_t + \sum_k \beta_k z_{k,n} + \varepsilon_{n,t}. \tag{A.2}$$

is estimated, where $z_{k,n}$ are quality characteristics of dwelling n (floor area, building age, location, etc.) and α_t are period fixed effects. The quality-adjusted price index is then $P_t^{\text{acq}} = \exp(\hat{\alpha}_t)/\exp(\hat{\alpha}_0)$. This is the method used in the main text.

Step 3: Apply the expenditure weight. The expenditure weight for OOH under the acquisitions approach is the value of net new purchases of dwellings in the base period,

$w^{\text{acq}} = P_0^H \cdot Q_0^{\text{net}}$, where Q_0^{net} is the net quantity of new dwellings acquired by households. The contribution of OOH to the overall CPI is $w^{\text{acq}} \cdot P_t^{\text{acq}}/P_0^{\text{acq}}$.

Step 4: Land inclusion or exclusion. A critical implementation decision is whether to include land in the price index. The Eurostat HICP framework excludes land (treating only the structure as a consumption good), so P_t^{acq} tracks construction costs only. If land is included, the hedonic approach in Step 2(b) is required to separate quality-adjusted dwelling prices from pure location effects.

Advantages

- **Conceptual simplicity.** No service-flow imputation is required (beyond standard quality adjustment), making the approach transparent and replicable.
- **Consistency with other durables.** The acquisitions approach treats housing symmetrically with all other durable goods in the CPI, such as automobiles and household appliances.
- **Based on actual transactions.** Prices are derived from market transactions, avoiding the subjective assumptions required for service-flow imputations.
- **Preferred by central banks for inflation monitoring.** Because it avoids imputation, the acquisitions index is free from assumptions about interest rates, depreciation, and expected capital gains, making it more robust for policy purposes;[5]

Disadvantages

- **Does not measure the service flow.** The acquisitions approach prices the *purchase* of a durable asset, not the *consumption* of shelter services. These two concepts coincide

[5] See Fenwick (2009, 2012) for this argument.

only for a one-period-lived asset. for long-lived dwellings, the acquisitions price embeds expectations about all future service flows and capital gains.

- **Highly volatile expenditure weights.** Net acquisitions fluctuate sharply over the housing cycle: they are large during booms and may collapse to near zero during downturns, even though households continue to consume shelter services from the existing stock. This makes the CPI sensitive to the choice of base year; see Hill, Steurer and Waltl (2017).

- **Understates the true cost of housing.** Because only new purchases are counted, the acquisitions weight is far smaller than the implicit rental value of the entire OOH stock. Studies consistently find that the acquisitions expenditure weight is 5–10 times smaller than the corresponding rental equivalence or user cost weight.

- **Problematic for cross-country comparisons.** In countries with high homeownership rates, the acquisitions weight is small because existing dwellings change hands infrequently. This makes international comparisons of inflation misleading when homeownership rates differ across countries.

- **Land decomposition difficulty.** If land is to be excluded (as in the HICP framework), the observed transaction price must be decomposed into land and structure components, which itself requires econometric methods.

A.2 The Rental Equivalence Approach

Theoretical Motivation

The rental equivalence approach imputes the period price of OOH services as the *market rent* for a comparable dwelling. The theoretical justification rests on the equilibrium condition that, in a competitive rental market, the rent a landlord charges just covers the full cost of providing one period of housing services, including the opportunity cost of capital. The System of National Accounts (1993) states:

> "As well-organized markets for rented housing exist in most countries, the output of own-account housing services can be valued using the prices of the same kinds of services sold on the market with the general valuation rules adopted for goods and services produced on own account. In other words, the output of housing services produced by owner-occupiers is valued at the estimated rental that a tenant would pay for the same accommodation, taking into account factors such as location, neighbourhood amenities, etc. as well as the size and quality of the dwelling itself."

Computation

Step 1: Choose the rent concept. A fundamental choice must be made between:

- **New-contract (market) rent** R_t^{new}: the rent agreed in a newly signed lease. This reflects the current opportunity cost of occupancy and is the theoretically correct concept for the Könus COLI.
- **Contract (roll-over) rent** R_t^{contract}: the rent paid by a continuing tenant under an existing lease. This is the concept used in most official CPIs, including Japan's, because it is easier to collect.

In a market with rising prices, $R_t^{\text{new}} > R_t^{\text{contract}}$ due to *rent stickiness*: landlords do not fully adjust rents for continuing tenants. The gap can be substantial and persistent over the housing cycle; see Shimizu, Nishimura and Watanabe (2010b), Genesove (2003), and Lewis and Restieaux (2015).

Step 2: Construct the quality-adjusted rent index. A hedonic regression is estimated on rental transaction data:

$$\ln R_{n,t} \;=\; \mu_t + \sum_k \beta_k^R \, z_{k,n} + \varepsilon_{n,t}^R, \tag{A.3}$$

where $z_{k,n}$ are the same quality characteristics as in the price hedonic (A.2). The quality-adjusted rent index is $P_t^{\text{rent}} = \exp(\hat{\mu}_t)/\exp(\hat{\mu}_0)$. A 13-month rolling-window version of this regression is used in the main text; see Appendix F for implementation details.

Step 3: Adjust for new-contract vs. contract rent. If contract rents are collected but new-contract rents are desired, a correction factor κ_t can be estimated as the ratio of new-contract to contract rent from a subsample of newly signed leases:

$$R_t^{\text{new}} \;=\; \kappa_t \cdot R_t^{\text{contract}}, \qquad \kappa_t \;\geq\; 1. \tag{A.4}$$

In Japan, κ_t has been estimated to exceed 1.3 during rapid appreciation episodes.

Step 4: Adjust for landlord-specific costs. Observed market rents include costs that do not apply to owner-occupiers: vacancy losses, property management fees, and insurance. Define ν_t as the vacancy and management rate (as a fraction of rental income) and π_t^{ins} as the insurance cost ratio. The imputed rent for an owner-occupier is

$$R_t^{\text{OOH}} \;=\; R_t^{\text{new}} \cdot (1 - \nu_t) - \pi_t^{\text{ins}} \cdot s_S \cdot P_t^H. \tag{A.5}$$

In practice, most statistical agencies skip this adjustment, so the rental equivalence measure slightly overstates the true OOH service cost; see the user cost equation in the main paper.

Step 5: Annualise and normalise. Monthly rents are converted to annual rates: $R_t^a = R_t^{\text{OOH}} \times 12$. The rent-to-price ratio is $u_t^R = R_t^a / P_t^H$. Normalise the index to the base year: $P_t^{\text{rent-idx}} = R_t^a / \bar{R}_{2020}^a \times 100$.

Advantages

- **Market-based.** Rents are observed in market transactions, providing a direct measure of what comparable shelter services cost without requiring assumptions about interest rates or depreciation.
- **Consistent with national accounts.** The SNA (1993) recommends the rental equivalence approach, ensuring consistency between the CPI and the national accounts measure of housing services.
- **Relatively smooth.** Contract rents adjust slowly, reducing CPI volatility. This is an advantage for monetary policy (which prefers smooth inflation signals) but a disadvantage for timeliness.
- **Widely used and internationally comparable.** Most OECD countries apply some form of rental equivalence, facilitating cross-country comparisons of inflation.

Disadvantages

- **Rent stickiness causes measurement lag.** Contract rents respond to market conditions with a substantial delay of 12–24 months. During rapidly appreciating markets, the rental equivalence index based on contract rents *understates* the current opportunity cost of OOH, giving a false sense of low inflation; see the empirical results in the main paper.
- **Comparability problem.** The rental and owner-occupied housing stocks differ systematically in size, quality, and location. Hedonic regression methods are required to

control for these differences, introducing estimation uncertainty.

- **No comparable rental market for high-end properties.** For luxury dwellings, there is often no rental market counterpart. Rents for high-end properties, when they exist, tend to be well below the properties' long-run user costs because only a limited pool of tenants can afford them. The rental equivalence approach thus *understates* the true cost of OOH for high-income households.

- **Includes landlord-specific costs.** Unless adjusted, observed rents overstate the true OOH cost by including vacancy and management costs not borne by owner-occupiers.

- **Requires imputation.** Despite being described as "market-based," the rental equivalence approach requires imputing a rent for properties that are not actually rented, introducing subjectivity.

A.3 The User Cost Approach

Theoretical Motivation

The user cost approach derives the period price of OOH services from the financial economics of asset ownership. It asks: what is the net cost to a household of purchasing a dwelling at the beginning of a period, occupying it for one period, and selling it at the end? This cost—the *user cost*—is the opportunity cost of deploying capital in a dwelling rather than in a financial instrument, net of the capital gain recovered at the end of the period. The approach is grounded in the capital theory of Jorgenson (1963) and Diewert (1974).

The user cost rate is defined by

$$u_t \;=\; r_t \;+\; c \;-\; \pi_t^e \tag{A.6}$$

where r_t is the nominal opportunity cost of capital, $c = \delta + \tau + m$ is the carrying cost rate, and π_t^e is the expected capital gain rate. The monetary user cost is $U_t = u_t \times P_t^H$. Financing-structure heterogeneity (Financial User Cost, FUC) is developed in Appendix E.

Step 1: Quality-Adjusted Housing Price Index

A hedonic price index P_t^H is constructed from transaction micro-data using equation (A.2). Normalise to the base year (P_t^H expressed with base-year average = 100).

Step 2: Depreciation Rate Measurement

The depreciation rate δ is one of the most critical parameters in the user cost formula. It determines how rapidly the structure component of a dwelling loses value, directly affecting both the carrying cost term $c = \delta + \tau + m$ and the land–structure decomposition of the user cost. Three approaches are commonly used to estimate δ:

(a) Cross-sectional age–price profile. Given secondhand transaction data at a point in time, estimate the relationship between observed price P_v^t and vintage v:

$$\ln P_v^t \; = \; \alpha_t + \beta v + \varepsilon \tag{A.7}$$

Under geometric depreciation, $\beta = \ln(1-\delta)$, so $\hat{\delta} = 1 - e^{\hat{\beta}}$. In practice, age enters the hedonic regression alongside structural characteristics (floor area, location, building type), and the coefficient on age identifies δ.

(b) Perpetual inventory method (PIM). If an estimate of the initial replacement cost $P_{S,0}$ of the structure is available, the current structure value after A years is $P_{S,A} = (1-\delta)^A P_{S,0}$. Matching observed resale prices against PIM-implied values yields an estimate of δ. The MLIT (Ministry of Land, Infrastructure, Transport and Tourism) construction cost surveys provide $P_{S,0}$ for Japan.

(c) National accounts benchmark. The Japanese national accounts report the aggregate depreciation rate for residential structures, which can be used as an external benchmark. The standard calibration in this study is $\delta = 0.020$ (annual rate, 2.0%), consistent with the MLIT surveys and the SNA estimate for reinforced concrete condominiums (Appendix C reviews the theoretical depreciation models; Table C.7 reports the empirical estimates from Tokyo hedonic regressions).

Sensitivity of user cost to δ. The user cost is linear in δ: $\partial u_t/\partial\delta = 1$. A one percentage-point increase in δ raises the user cost rate by exactly one percentage point, independent of interest rates or capital gains. For a dwelling with $P_t^H = 100$ (index units), this translates to a change in the monetary user cost U_t of one index unit per year. Uncertainty about δ is therefore a first-order source of uncertainty about the level of the user cost index.

Step 3: Expected Capital Gain Rate and the Volatility Problem

The expected capital gain rate π_t^e is the most *volatile and empirically challenging* parameter in the user cost formula. This section explains why, and how smoothing addresses the problem.

(a) The realised rate and its volatility. The ex-post (realised) one-period capital gain rate is

$$\pi_t^{\text{act}} = \frac{P_t^H - P_{t-1}^H}{P_{t-1}^H} \times 12 \tag{A.8}$$

(annualised from monthly data). Substituting π_t^{act} for π_t^e in (A.6) gives the *Basic User Cost*:

$$u_t^{\text{basic}} = r_t^{\text{prime}} + c - \pi_t^{\text{act}} \tag{A.9}$$

The Basic UC inherits the full volatility of π_t^{act}, which is large for the same reason that the acquisitions index is volatile: housing asset prices respond immediately to changes in market expectations, whereas the flow cost of shelter services moves slowly. In fact, Basic UC and the acquisitions price index are driven by the same underlying process: both embed current-period capital gains directly. The key difference is that in Basic UC, the capital gain enters with a *negative* sign (it offsets the cost of occupancy), so periods of rapid price appreciation—such as the Tokyo bubble of 1987–1990 and the post-COVID period of 2021–2025—can drive u_t^{basic} sharply *negative*.

(b) The problem of negative user costs. A negative user cost ($u_t < 0$) implies that the financial gain from holding the dwelling exceeds all carrying costs, so living in the dwelling appears effectively costless in accounting terms and may even generate net income. While this is a theoretically coherent result—it occurs in genuine bubble episodes when the investment motive dominates—it is problematic as a price concept for the CPI for two reasons:

(i) A negative price cannot be averaged meaningfully with other positive CPI prices in a standard index number formula.

(ii) The signal is driven by *expected* future gains that may not materialise. If the bubble bursts, the realised gain is reversed, but the index has already recorded negative service-flow costs.

These problems motivate the use of a *smoothed* expected appreciation rate that captures medium-run trends rather than short-run fluctuations.

(c) HP-filter smoothing. The standard approach, due to Hodrick and Prescott (1997) and calibrated for monthly data by Ravn and Uhlig (2002), applies the HP filter to the log price series $\ell_t = \ln P_t^H$:

$$\min_{\{\tilde{\ell}_t\}} \underbrace{\sum_t \left(\ell_t - \tilde{\ell}_t\right)^2}_{\text{fit}} + \bar{\lambda}_{\text{HP}} \underbrace{\sum_t \left(\tilde{\ell}_{t+1} - 2\tilde{\ell}_t + \tilde{\ell}_{t-1}\right)^2}_{\text{smoothness}} \tag{A.10}$$

with $\bar{\lambda}_{\text{HP}} = 129{,}600$ (monthly). The smoothed expected appreciation rate is

$$\pi_t^e = \left(\tilde{\ell}_t - \tilde{\ell}_{t-1}\right) \times 12 \tag{A.11}$$

The HP filter decomposes ℓ_t into a slowly-moving trend $\tilde{\ell}_t$ and a cyclical component. By construction, π_t^e tracks the medium-run trend in housing prices rather than month-to-month fluctuations, substantially reducing—though not eliminating—the volatility of the resulting user cost.

(d) Alternative smoothing methods. The HP filter is not the only option. Alternative approaches include:

- *Rolling-window average:* $\pi_t^e = \frac{1}{H}\sum_{h=1}^{H} \pi_{t-h}^{\text{act}}$ for a window of H months (typically $H = 60$ or 120). This method is backward-looking and does not impose a smoothness penalty on the trend; it tends to respond more slowly to turning points.
- *Long-run geometric average:* $\pi_t^e = [(P_t^H / P_{t-K}^H)^{1/K} - 1] \times 12$ for a window of K years (15–25 years is typical). Diewert and Fox (2018) advocate this method for land price inflation.
- *Rational expectations approximation:* Set π_t^e equal to a constant long-run growth rate estimated over the full sample, reflecting the view that rational households form expectations based on secular trends rather than recent fluctuations.

All three alternatives yield smoother series than Basic UC but differ in their responsiveness to regime changes. The sensitivity of the empirical results to the choice of smoothing method is documented in the main paper.

(e) Summary: the smoothing trilemma. There is an inherent tension between three desiderata for π_t^e: (1) *accuracy* (close to the true expected gain), (2) *timeliness* (responsive to current market conditions), and (3) *stability* (free from excessive volatility). No single method achieves all three simultaneously: the Basic UC maximises timeliness at the cost of stability; a long rolling average maximises stability at the cost of timeliness; the HP filter provides an intermediate balance. The choice of smoothing method is therefore a substantive empirical decision, not merely a technical one, and it materially affects the measured CPI implications of the user cost approach.

Step 4: Carrying Cost Rate

The total annual carrying cost rate is

$$c \;=\; \delta + \tau + m \;=\; 0.020 + 0.014 + 0.000 \;=\; 0.034 \tag{A.12}$$

where $\delta = 0.020$ is the annual structure depreciation rate, $\tau = 0.014$ is the property tax rate (Japan's standard rate under the Local Tax Law), and $m = 0$ is the maintenance cost rate (set to zero in the baseline calibration).

Step 5: Basic User Cost Rate and Value

The Basic User Cost (introduced here; the FUC extensions are treated in Appendix E) uses the prime rate as the financing cost and, optionally, either the realised or HP-smoothed appreciation rate:

$$u_t^{\text{basic,act}} \;=\; r_t^{\text{prime}} + c - \pi_t^{\text{act}} \tag{A.13}$$

$$u_t^{\text{basic,HP}} \;=\; r_t^{\text{prime}} + c - \pi_t^{e} \tag{A.14}$$

The monetary user cost and index are

$$U_t^{\text{basic}} = u_t^{\text{basic}} \times P_t^H \tag{A.15}$$

$$P_t^{\text{UC,basic}} = U_t^{\text{basic}} \Big/ \bar{U}_{2020}^{\text{basic}} \times 100 \tag{A.16}$$

Step 6: Land–Structure Decomposition (Optional)

For a more precise user cost, the land and structure components are valued separately, since depreciation applies only to the structure. Using the hedonic decomposition of Appendix D, the full user cost rate is

$$u_t = (r_t - i_{Lt})\, s_L + \left[r_t - i_{St} + (1 + i_{St})\delta\right] s_S + \tau \tag{A.17}$$

where s_L, s_S are land and structure value shares, and i_{Lt}, i_{St} are their expected appreciation rates. In the baseline empirical analysis we use the simplified single-rate formula (A.14), which applies one HP-smoothed rate to total property value. The FUC variants that incorporate financing-structure heterogeneity are developed in Appendix E.

Advantages

- **COLI-consistent.** Directly measures the period-by-period cost of shelter services, satisfying the Könus constant-utility benchmark.
- **Responds to current market conditions.** When interest rates rise sharply, the user cost rises immediately without the lag of contract rent adjustment.
- **Captures capital gains correctly.** The financial benefit of holding an appreciating asset is properly netted out from the occupancy cost.
- **Applicable where rental markets are thin.** The only feasible approach for high-

end dwellings with no comparable rental market.

Disadvantages

- **Negative user costs during bubbles.** The Basic UC (using realised appreciation) turns negative when capital gains exceed carrying costs. Smoothing mitigates but does not fully resolve this problem.
- **High volatility shared with the acquisitions approach.** Both the acquisitions index and the Basic UC are driven by short-run asset price fluctuations. The FUC variants (Appendix E) address this via HP smoothing and leverage-weighted financing rates.
- **Sensitive to parameter assumptions.** Small changes in δ, τ, or the smoothing window for π_t^e materially affect index levels.
- **The smoothing trilemma.** No single method for estimating π_t^e achieves simultaneously accuracy, timeliness, and stability; see paragraph (e) above.
- **Difficult to communicate.** Less intuitive than market rent; national statistical agencies have been slow to adopt it (Iceland is the notable exception).

A.4 Alternative Approaches

This section describes two additional approaches that have been used to measure OOH services: the payments approach (Section A.4.1), which measures actual cash outflows, and the opportunity cost approach (Section A.4.2), which takes the maximum of the user cost and rental equivalence as the theoretically correct service-flow price.

A.4.1 The Payments Approach

A further approach to the treatment of owner-occupied housing in the CPI, the *payments approach*, is described by Goodhart as follows:

> "The second main approach is the payments approach, measuring actual cash outflows, on down payments, mortgage repayments and mortgage interest, or some subset of the above. ... Despite its problems, such a cash payment approach was used in the United Kingdom until 1994 and still is in Ireland." Charles Goodhart (2001, pp. F350–F351).

Thus the *payments approach* to owner-occupied housing is a kind of *cash flow approach* to the costs of operating an owner-occupied dwelling. It consists mainly of mortgage interest and principal payments along with property taxes. Imputations for capital gains, for the cost of capital tied up in house equity and depreciation are ignored in this approach. This leads to the following objections to this approach; i.e., it ignores the opportunity costs of holding the equity in the owner-occupied dwelling, it ignores depreciation and it uses nominal interest rates without any offset for inflation in the price of land and the structure. In general, the payments approach will tend to lead to much smaller monthly expenditures on owner occupied housing than the other 4 main approaches, except during periods of high inflation, when the nominal mortgage rate term may become very large without any offsetting item for inflation. One reason for implementing this approach is that it may be useful for indexing the pensions of homeowners; i.e., as the cash costs of home ownership increase, it may be popular to increase pensions to offset these costs.[6] This line of argument has some validity but in recent years, perhaps it is less compelling in many countries due to the ability of homeowners to draw on their equity with reverse mortgages and to postpone paying property taxes until the property is sold. However, a cash flow or payments approach to the valuation of the

[6] Thus the UK still uses the payments approach to value OOH in its Retail Prices Index.

costs of home ownership may be useful for some users.[7]

In the following paragraphs, we outline in more detail under what circumstances it may be useful to use the payments approach in the CPI.[8]

The payments approach is more consistent with the traditional approach to CPI construction, which is a carry-over from a time when the CPI was mostly used as a compensation tool. It also has much to commend it from the point of view of public acceptability as changes in the cost of servicing the interest on loans that have been taken out to purchase owner-occupier housing, following changes in interest rates, are reflected in the index. Home owners who face real increases in costs from a rate rise, for instance, would expect to see this reflected in their cost of living as measured by an official CPI. A user cost approach would reflect this increase in costs but a rental equivalence approach may not reflect the magnitude of the increase in interest rates. It is counter-intuitive to home owners that under the "rental equivalence" approach owner occupier housing costs can increase when house prices are falling and interest rates are stable and more people are looking to live in rent accommodation thereby increasing rents. The latter happened in the UK following the financial crisis of 2008.[9]"Mortgage interest" is more likely to be understood by the general public than "rental equivalence" and, unlike the latter, the index will directly reflect changes in house prices and interest rates.

From a technical viewpoint the payments approach measures the following direct expenditures by owners of property: mortgage interest payments; repayment of capital; large repairs (associated with depreciation from wear and tear as properties get older). There are no imputed prices. Index compilers typically take every effort to avoid the latter in other

[7]Fenwick (2009, 2012) has argued strongly that statistical agencies responsible for consumer price indexes should produce a range of indexes that suit different purposes. Thus the payments approach to OOH could be produced by statistical agencies that provide multiple consumer price indexes to suit different purposes. However, the payments approach cannot serve as a reliable guide for pricing the services of OOH.

[8]he following four paragraphs are due to David Fenwick.

[9]The user cost approach could also generate an increase in OOH under these circumstances: expected rates of property inflation could fall, which would lead to an increase in user costs.

expenditure categories in a CPI.

As with other approaches to the measurement of owner-occupier housing costs there are some problems associated with the payments approach. A CPI should only relate to consumption items and should exclude cash disbursements or expenditures which are in the nature of savings or "investments". The acquisition of a house can represent a substantial capital asset – a point which emerges when comparing the position of owner-occupiers with tenants who rent. It can therefore be argued that the capital element of mortgage repayments should be regarded as an investment or saving rather than consumption expenditure and should therefore be excluded from the index. But the counter-argument is that the capital gains of owning a house which appreciates in value, is of limited relevance when people have to bear such costs from current income. The question also arises over whether the weight and price indicator should be net of any tax allowances for mortgage interest payments. It is in accordance with the principle that a CPI should be based on the amounts actually paid, that the weight and price indicator should both be based on payments after tax relief. The biggest practical barrier to the adoption of the payments approach is that it requires a large volume of data, which may or may not be available to the compiler. The calculation of the price indicator for interest charges involves two components: the rate of interest and the average amount of mortgage debt outstanding. To calculate the average outstanding debt at any one point in time can be problematic as it consists of a large number of individual debts, some from mortgages taken out recently and others from mortgages taken out some time ago at historic prices and with some of the debt paid back. Consistent with the fixed basket approach, average payments are calculated with respect to a fixed stock of new and existing mortgages (of various ages) equivalent to those existing in the base period. Finally, it can be noted that the inclusion of owner-occupier housing costs using the payments approach can increase the volatility of the index in periods where interest rates frequently change.

To conclude this discussion of the possible uses of the payments approach, we note that the approach chosen should align as closely as possible with the conceptual basis which best satisfies the principal purpose of the CPI, subject to adequate data being available to make a reliable calculation. This applies, of course, to all expenditures covered in a CPI.

At this point, it is useful to review the three ways which can be used to measure consumption expenditures. The following quotation from the Office for National Statistics (2010, p. 6):

> "Consumption expenditure can be measured in three ways which it is important to distinguish. These ways are:
>
> *Acquisition* means that the total value of all goods and services delivered during a given period is taken into account, whether or not they were wholly paid for during the period.
>
> *Use* means that the total value of all goods and services consumed during a given period is taken into account.
>
> *Payment* means that the total payments made for goods and services during a given period is taken into account, whether or not they were delivered.
>
> For practical purposes, these three concepts cannot be distinguished in the case of non-durable items bought for cash, and they do not need to be distinguished for many durable items bought for cash. The distinction is, however, important for purchases financed by some form of credit, notably major durable goods, which are acquired at a certain point of time, used over a considerable number of years, and paid for, at least partly, some time after they were acquired, possibly in a series of installments. Housing costs paid by owner-occupiers are an obvious example."

In what follows, we will look at the problems associated with the three methods of valuation in a number of specific cases.

Case 1: The payment period coincides with the acquisition period. Let P_1 be the acquisition price for such a unit of a durable good in period 1. Then the acquisition price in period

1 is obviously P_1, the payments price is also P_1 and the period 1 user cost price is p_1 and its exact form depends on the model of depreciation that is applicable for this particular durable good. In other words, there are no problems in sorting out the three methods of valuation in this case.

Case 2: The initial payment period coincides with the acquisition period but payments for the purchase of the durable continue on for subsequent periods. Suppose that payments must be made for T periods and the sequence of monetary payments is $\pi_1, \pi_2, \ldots, \pi_T$. Suppose also that the sequence of expected one period financial opportunity costs of capital for the purchasing household is $r_1, r_2, \ldots, r_{T-1}$. Then the discounted stream of payments, P_1, is the period 1 (expected) cost of purchasing the good where P_1 is defined as follows:

$$P_1 \equiv \pi_1 + (1+r_1)^{-1}\pi_2 + (1+r_1)^{-1}(1+r_2)^{-1}\pi_3 + \cdots + (1+r_1)^{-1}(1+r_2)^{-1} \cdots (1+r_{T-1})^{-1}\pi_T. \quad \text{(A.18)}$$

In this case, the acquisitions price for the durable good in period 1 is defined to be P_1, the payments price is π_1 and the user cost will be determined using the appropriate depreciation model where P_1 is taken to be the beginning of the period price for the durable good. In a subsequent period $t \leq T$, the acquisitions price for the used durable good will be 0, the payments price will be π_t and the period t user cost value v_t will be determined using the appropriate depreciation model for this type of durable good. *If* the useful life of the durable good happens to equal T and *if* the period t payment is equal to the corresponding period t user cost valuation v_t for $t = 1, 2, \ldots, T$, then obviously, the period t user cost valuation v_t will be equal to the observable period t payment π_t.[10]

There are problems associated with the computation of the P_1 defined by (A.18); i.e., in

[10] The period t user cost valuation v_t for a unit of the durable good that is t periods old can be converted into an equivalent amount of a new unit of a durable good if the geometric or one hoss shay model of depreciation is applicable for the durable good under consideration. Otherwise, units of the durable good of different ages at the same point in time need to be aggregated using an index number formula.

order to compute P_1 when the durable good is purchased during period 1, the sequence of future payments π_t has to be known and guesses will have to be made on the magnitudes of the sequence of expected nominal interest rates r_t. However, the important point to be made here is that P_1 defined by (A.18) will be *less* than the simple sum of the π_t, $\sum_{t=1}^{T} \pi_t$, provided that the nominal interest rates r_t are positive.

Case 3: *The full payment for the good (or service) is made in period 1 but the services of the commodity are not delivered until period t.* Let the period 1 payment be π_1 as usual. Thus the sequence of payments associated with the purchase of the commodity under consideration is π_1 for period 1 and 0 for all subsequent periods. The acquisition of the commodity does not take place until period t but the appropriate acquisition price P_t is not the period 1 payment, π_1, but the following *escalated period 1 price*:

$$P_t \equiv (1 + r_1)(1 + r_2) \cdots (1 + r_{t-1})\pi_1. \tag{A.19}$$

The logic behind this valuation is the following one. During period 1 when the product was paid for, the payment could have been used to pay down debt (at the interest rate r_1) or the payment could have been used to invest in an asset that earned the rate of return r_1. Thus after one period, the opportunity cost of the investment in the pre-purchased product has grown to $\pi_1(1+r_1)$, after 2 periods, the opportunity cost has grown to $\pi_1(1+r_1)(1+r_2)$,. . . , and by period t when the good or service is acquired, the opportunity cost has grown to $\pi_1(1 + r_1)(1 + r_2) \cdots (1 + r_{t-1})$, which is (A.19). The important point to be made here is that P_t defined by (A.19) will be *greater* than the period 1 prepayment, π_1, provided that the nominal interest rates r_t are positive. Since the product has not been acquired by the household for periods $1, 2, \ldots, t-1$, the corresponding user cost valuations, $v_1, v_2, \ldots, v_{t-1}$ should be set equal to 0. However, when period t is reached, "normal" user costs can be calculated for durable goods using the P_t defined by (A.19) as the beginning of period t price of the durable, assuming that the form of depreciation is known.

Prepayment for services or durable goods is widespread; e.g., trip and hotel reservations made in advance and paid for in advance are service examples and prepayment for condominium units that are under construction is a durable good example.

Case 4: The good or service is acquired in period 1 but is not paid for until period 2. In this case, the sequence of payments is $0, \pi_2, 0, \ldots, 0$. The commodity is acquired in period 1 and the appropriate period 1 acquisition price is P_1 defined as follows:

$$P_1 \equiv (1 + r_1)^{-1}\pi_2. \tag{A.20}$$

The justification for this acquisition price runs as follows: The purchasing household lays aside the amount of money P_1 to buy the product in period 1. This money is invested and earns the one period rate of return r_1. Thus when period 2 comes along, the household has $P_1(1 + r_1) = \pi_2$ which is just enough money to complete the purchase in period 2. Thus P_1 is an appropriate period 1 acquisitions price. If the commodity is a durable good, then assuming that the form of depreciation is known, P_1 defined by (A.20) can be used as the beginning of period 1 price for the period 1 user cost and the entire sequence of user costs can be calculated.

This form of pricing is used as a way of offering lower prices for a wide variety of products. A particular application of this model to a service is the use of credit cards to purchase consumption items. A household that pays its balance owing on time can avoid interest charges and thus can postpone payment for its household purchases for up to one month in many cases.[11]

If interest rates are very low, then statistical agencies may well find it is not worth taking into account the above refinements. However, if nominal interest rates are high, it may be necessary to make some of the above adjustments.

[11] However, a household that does not pay off its balance owing in a timely fashion will find itself in Case 3 above.

A.4.2 The Opportunity Cost Approach

The opportunity cost approach to the valuation of OOH services was first proposed by Diewert (2008) and further developed by Diewert and Nakamura (2011). The central insight is that a household owning a dwelling foregoes *two* distinct opportunities: (i) renting the property to a tenant at the prevailing new-contract rent R_t^{new}, and (ii) liquidating the property and investing the proceeds at the financial user cost FUC_t. The true opportunity cost of owner-occupancy is the maximum of these two foregone alternatives:

$$\text{OC}_t \;=\; \max\bigl\{R_t^{\text{new}},\; U_t\bigr\} \tag{A.21}$$

where $U_t = u_t \cdot P_t^H$ is the monetary user cost value (see equation (A.15) in Appendix A.3).

Computation.

1. Compute the quality-adjusted new-contract rent R_t^{new} using the hedonic rent regression described in Appendix A.2.
2. Compute the user cost value U_t using any of the FUC variants (A, B, or C) described in Appendix A.3.
3. Take the element-wise maximum: $\text{OC}_t = \max\{R_t^{\text{new}},\ U_t\}$.
4. The opportunity cost price index is $P_t^{\text{OC}} = \text{OC}_t/\bar{\text{OC}}_{2020} \times 100$.

Regime interpretation. The opportunity cost measure shifts endogenously between two regimes:

- **Investment-dominant regime** ($U_t < R_t^{\text{new}}$, i.e., $u_t < R_t^{\text{new}}/P_t^H$): the dwelling's financial opportunity cost is low (or negative) relative to the rent it could earn. This occurs when expected capital gains are large, driving down the net cost of ownership.

In this regime, $\mathrm{OC}_t = R_t^{\mathrm{new}}$ and the opportunity cost measure coincides with the rental equivalence measure. Tokyo exhibited this regime during the bubble period (1987–1990) and the recent appreciation episode (2021–2025).

- **Capital-cost dominant regime** ($U_t > R_t^{\mathrm{new}}$): the user cost exceeds the achievable rent. This occurs when interest rates are high or expected capital gains are low. In this regime, $\mathrm{OC}_t = U_t$ and the opportunity cost measure coincides with the user cost measure. Tokyo exhibited this regime during the high-interest-rate period (1989–1993) and the deflation era (1993–2003).

Advantages.

- **True opportunity cost.** The approach correctly measures what the household sacrifices by owner-occupying rather than pursuing the best available alternative, thus satisfying the COLI requirement (see Online Appendix B.1).
- **Regime-adaptive.** Unlike the user cost or rental equivalence approaches individually, the opportunity cost measure automatically selects the relevant binding constraint, providing a more stable and economically meaningful index across different market environments.
- **Always non-negative.** Since $\mathrm{OC}_t \geq R_t^{\mathrm{new}} > 0$, the opportunity cost measure avoids the negative-value problem that afflicts the user cost approach during bubble episodes.
- **Upper bound on OOH service cost.** $\mathrm{OC}_t \geq U_t$ and $\mathrm{OC}_t \geq R_t^{\mathrm{new}}$, so the opportunity cost approach provides an upper bound on the true COLI price, closing the gap that arises under the rental equivalence or user cost approach individually.

Disadvantages.

- **Requires both a rent index and a user cost.** The approach is more data-intensive than either the rental equivalence or user cost approach individually.

- **Unfamiliar to practitioners.** Despite its theoretical appeal, the opportunity cost approach has been implemented empirically only in a small number of studies;[12] national statistical agencies have not yet adopted it.

- **Discrete regime switching.** At the boundary where $U_t = R_t^{\text{new}}$, the opportunity cost index may exhibit a kink, introducing apparent discontinuities in measured CPI inflation.

[12] Shimizu, Diewert, Nishimura and Watanabe (2012) for Tokyo; Aten (2018) for the United States.

B Theoretical Relationships among the Three Approaches under the Cost-of-Living Index Framework

This appendix provides a rigorous derivation of the relationships among the three main OOH measurement approaches—acquisitions, rental equivalence, and user cost—within the framework of the Könus cost-of-living index (COLI). Section B.1 establishes the COLI benchmark for a housing durable. Section B.2 derives the equilibrium and disequilibrium relationships between the user cost and rental equivalence. Section B.4 derives the relationship between the user cost and acquisitions approaches, including an explicit formula for the ratio of expenditure values under the two approaches. Section B.5 summarises the theoretical ordering of all three approaches under the COLI criterion.

B.1 The COLI Benchmark for a Consumer Durable

Let the household's preferences over non-durable consumption $\mathbf{q}^{\text{nd}}$ and housing services h be represented by a utility function $U(\mathbf{q}^{\text{nd}}, h)$, where h denotes one period's worth of constant-quality shelter. The expenditure function is $e(\mathbf{p}^{\text{nd}}, p^h, u) \equiv \min\{(\mathbf{p}^{\text{nd}} \cdot \mathbf{q}^{\text{nd}}) + p^h \cdot h : U(\mathbf{q}^{\text{nd}}, h) \geq u\}$, where p^h is the *period price of one unit of housing services.* The Könus COLI between periods 0 and t is

$$P^K \equiv \frac{e(\mathbf{p}^{t,\text{nd}},\, p^{h,t},\, u^*)}{e(\mathbf{p}^{0,\text{nd}},\, p^{h,0},\, u^*)} \tag{B.1}$$

The COLI is *well-defined* if and only if $p^{h,t}$ is a well-defined scalar price for one period's occupancy of a constant-quality dwelling. For non-durables, this is trivially satisfied. For housing—a good that delivers services over many periods—the key question is: *which of the three approaches yields a price $p^{h,t}$ that is consistent with the COLI?*

The service-flow requirement. The Könus COLI requires that $p^{h,t}$ represent the *minimum expenditure per period* needed to obtain one unit of constant-quality housing services in period t, given that the household can choose optimally between owning and renting. More precisely, let $p^{h,t}_{\text{own}}$ be the cost of owner-occupancy and $p^{h,t}_{\text{rent}}$ the cost of renting. In a market without frictions, the household selects the cheaper option, so the COLI-consistent price is

$$p^{h,t}_{\text{COLI}} \;=\; \min\left\{p^{h,t}_{\text{own}},\; p^{h,t}_{\text{rent}}\right\} \tag{B.2}$$

In Section B.2 we show that in competitive equilibrium $p^{h,t}_{\text{own}} = p^{h,t}_{\text{rent}}$, so the COLI price reduces to either the user cost or the market rent. In disequilibrium, (B.2) implies that the *lower* of the two—the user cost when it is below rent, the rent when it is above—is the COLI-consistent price. This is precisely the *opportunity cost approach* proposed by Diewert (2008).

B.2 User Cost and Rental Equivalence: Equilibrium and Disequilibrium

Competitive equilibrium condition

Consider a risk-neutral landlord who purchases a dwelling at price P^H_t at the beginning of period t, rents it out at rate R_t during the period, and sells it at price $P^H_{t+1}(1-\delta)$ at the end of the period (where δ is the structure depreciation rate and we set the land share to unity for simplicity; see Section B.3 below for the general case). The landlord's one-period profit (per unit of capital) is

$$\Pi_t \;=\; R_t + P^H_{t+1}(1-\delta) - P^H_t(1+r_t) \tag{B.3}$$

where r_t is the landlord's cost of capital. In a competitive market with free entry, profits are driven to zero: $\Pi_t = 0$, which gives

$$R_t^* \;=\; P_t^H(1 + r_t) - P_{t+1}^H(1 - \delta) \;=\; U_t \tag{B.4}$$

where $U_t = [(1 + r_t) - (1 - \delta)(1 + \pi_t^e)]P_t^H$ is the end-of-period user cost defined in Appendix A.3, equation (A.6). Equation (B.4) is the fundamental equilibrium condition: *in long-run competitive equilibrium, market rent equals the user cost.* This establishes that the rental equivalence and user cost approaches are theoretically equivalent—both provide a consistent measure of the COLI-implied housing price—when equilibrium holds.

Sources of disequilibrium

In practice, several frictions cause $R_t \neq U_t$. Introducing vacancy costs ν_t, maintenance expenditure m_t, insurance π_t^{ins}, and distinguishing the depreciation rates for rented properties (δ^R) and owner-occupied properties (δ^O), the landlord's zero-profit condition becomes

$$R_t \;=\; \left[r_t - \pi_t^e + \delta^R(1 + \pi_t^e) + \tau + m_t + \nu_t + \pi_t^{\text{ins}}\right] P_t^H \tag{B.5}$$

while the owner-occupier's user cost is

$$u_t^O \;=\; r_t - \pi_t^e + \delta^O(1 + \pi_t^e) + \tau \tag{B.6}$$

The gap between the observed rent and the owner-occupier's user cost rate is therefore

$$\frac{R_t}{P_t^H} - u_t^O \;=\; (\delta^R - \delta^O)(1 + \pi_t^e) + m_t + \nu_t + \pi_t^{\text{ins}} \;\;\geq\;\; 0 \tag{B.7}$$

Equation (B.7) decomposes the rent–user cost gap into four identifiable components:

1. $(\delta^R - \delta^O)(1 + \pi_t^e)$: the *depreciation differential.* Landlords maintain properties less

carefully than owner-occupiers ($\delta^R > \delta^O$), so the quality of a rented dwelling declines faster. For Tokyo condominiums, Shimizu et al. 2012 estimate $\delta^R \approx 0.030$ versus $\delta^O \approx 0.020$.

2. m_t: landlord maintenance and overhead costs.

3. ν_t: vacancy and management premium.

4. π_t^{ins}: insurance costs.

Since all four components are non-negative under normal conditions, (B.7) implies that *observed market rent is a systematically upward-biased proxy for the owner-occupier's user cost.* The rental equivalence approach therefore overstates the COLI-consistent price of OOH by the sum of these frictions.

The stickiness wedge

An additional source of divergence arises from rent stickiness. Let R_t^{new} be the new-contract rent and R_t^{contract} the roll-over (contract) rent. In a market with rising prices, $R_t^{\text{new}} > R_t^{\text{contract}}$ because landlords do not fully adjust rents for continuing tenants. The COLI-consistent price requires R_t^{new} (the current opportunity cost of shelter), whereas most CPIs measure R_t^{contract} (a weighted average of old and new contracts). Define the stickiness wedge as

$$\sigma_t \;\equiv\; R_t^{\text{new}} - R_t^{\text{contract}} \;\geq\; 0 \tag{B.8}$$

During appreciation episodes, σ_t is large and positive, causing official CPIs that use contract rents to *understate* the current opportunity cost of OOH. Conversely, during price declines, σ_t can be negative, causing official CPIs to *overstate* the true cost. The empirical magnitude of σ_t in Tokyo is documented in the empirical analysis of the main paper.

B.3 Land and Structure Components

A dwelling consists of two components with fundamentally different economic properties: *land* (which does not depreciate) and *structure* (which depreciates at rate δ). Let $V_t = P_{Lt}L + P_{St}(1-\delta)^A S$ be the total property value, where P_{Lt} and P_{St} are the land and structure prices, L is land area, S is floor space, and A is building age. Define $s_L \equiv P_{Lt}L/V_t$ and $s_S \equiv P_{St}(1-\delta)^A S/V_t$ as the land and structure value shares.

The full user cost rate for land and structure separately is

$$u_t \;=\; (r_t - i_{Lt})\, s_L \;+\; \big[r_t - i_{St} + (1+i_{St})\delta\big]\, s_S \;+\; \tau \tag{B.9}$$

where i_{Lt} and i_{St} are the expected land and structure price appreciation rates. Since i_{Lt} typically exceeds i_{St} (land appreciates faster than structures in long-run urban equilibrium), the land component of the user cost is lower than the structure component ceteris paribus.

The landlord's full rent decomposition analogous to (B.5) is

$$\begin{aligned}\frac{R_t}{V_t} \;&=\; [r_t - i_{Lt} + \nu_t + \tau_L]\, s_L \\ &\quad+\; [r_t - i_{St} + (1+i_{St})\delta^R + \nu_t + \tau_S + m_t + \pi_t^{\text{ins}}]\, s_S \end{aligned} \tag{B.10}$$

where τ_L and τ_S are the property tax rates on land and structure. Subtracting (B.9) from (B.10), the rent–user cost gap becomes

$$\frac{R_t}{V_t} - u_t \;=\; \nu_t \;+\; \big[(\delta^R - \delta^O)(1+i_{St}) + m_t + \pi_t^{\text{ins}}\big]\, s_S \;\;\geq\;\; 0 \tag{B.11}$$

The gap applies only to the *structure share* s_S for the depreciation and maintenance components, plus the overall vacancy premium. This refines the earlier result in (B.7).

B.4 User Cost and Acquisitions: Value Ratio

The acquisitions approach and the user cost approach differ fundamentally in what they measure: the former prices *new purchases* of dwellings, while the latter prices the *service flow from the entire stock* of dwellings. This section derives, following Diewert (2002), an explicit formula for the ratio of the two expenditure values.

Setup. Assume geometric depreciation at rate δ, constant real interest rate r^*, and that new dwelling purchases have grown at the geometric rate g into the indefinite past. The *acquisitions value* in period 0 is

$$V_A^0 \;\equiv\; P^0 q^0 \tag{B.12}$$

The *user cost value* in period 0 is the sum of user costs over all vintages of dwellings currently in use:

$$\begin{aligned} V_U^0 \;&\equiv\; p^0 q^0 + (1-\delta)p^0 \frac{q^0}{1+g} + (1-\delta)^2 p^0 \frac{q^0}{(1+g)^2} + \cdots \\ &=\; p^0 q^0 \sum_{v=0}^{\infty} \left(\frac{1-\delta}{1+g}\right)^v \;=\; \frac{(1+g)\,p^0\,q^0}{g+\delta} \end{aligned} \tag{B.13}$$

where the sum converges provided $g < \delta + g$, i.e., $\delta > 0$. Substituting the user cost formula $p^0 = (r^* + \delta)P^0$ (the simplified end-of-period user cost with zero asset inflation), the ratio of the two value flows is

$$\frac{V_U^0}{V_A^0} \;=\; \frac{(1+g)(r^*+\delta)}{g+\delta} \tag{B.14}$$

Properties of the ratio. Equation (B.14) has several important implications:

1. **The ratio exceeds unity under normal conditions.** If $r^* > g(1-\delta)/(1+g)$—a condition satisfied whenever the real interest rate exceeds the depreciation-adjusted growth rate of housing investment—then $V_U^0/V_A^0 > 1$. The acquisitions approach therefore *understates* the true value of housing consumption.

2. **The ratio is increasing in r^*.** $\partial(V_U^0/V_A^0)/\partial r^* = (1+g)/(g+\delta) > 0$. Higher real interest rates amplify the gap between the user cost and acquisitions measures.

3. **The ratio is decreasing in δ.** $\partial(V_U^0/V_A^0)/\partial \delta = -(1+g)r^*/(g+\delta)^2 < 0$. For long-lived assets (small δ), the gap is large. for short-lived assets (large δ), the two approaches converge.

4. **Numerical calibration for housing.** Using annual values $\delta = 0.02$, $r^* = 0.03$, $g = 0.01$:

$$\frac{V_U^0}{V_A^0} = \frac{1.01 \times 0.05}{0.03} \approx 1.68 \tag{B.15}$$

   The user cost measure values housing consumption at *1.68 times* the acquisitions measure. If the depreciation rate is halved to $\delta = 0.01$, the ratio rises to 2.02; if the real interest rate is raised to $r^* = 0.04$, the ratio rises to 2.02. These calibrations confirm that the acquisitions approach systematically and substantially *understates* the weight that housing should receive in the CPI.

5. **Convergence for short-lived durables.** For clothing or appliances with $\delta = 0.20$, $r^* = 0.03$, $g = 0.01$: $V_U^0/V_A^0 \approx 1.11$. The two approaches nearly coincide, justifying the acquisitions approach for high-depreciation-rate goods.

Asset price as present value of user costs. A complementary perspective on the acquisitions–user cost relationship comes from the asset pricing identity. Under geometric depreciation and constant parameters, the current asset price equals the present discounted value of all future user costs:

$$P_t^H = \sum_{s=0}^{\infty} \frac{(1-\delta)^s \, u_{t+s} \, P_{t+s}^H}{(1+r)^s} \tag{B.16}$$

In steady state with constant growth rate π^e of asset prices, this simplifies to

$$P_t^H = \frac{u_t P_t^H}{r - \pi^e + \delta} \implies u_t = r - \pi^e + \delta = r^* + \delta \tag{B.17}$$

confirming the simplified user cost formula. Crucially, (B.16) reveals that the acquisitions price P_t^H encodes *expectations about all future* user costs and capital gains, not merely the current period. A shock to expected future interest rates or depreciation rates therefore shifts the acquisitions index immediately, while the user cost index responds only gradually. This is the fundamental reason why the acquisitions index is far more volatile than the user cost index.

B.5 Theoretical Ordering under the COLI

The analysis above establishes the following ordering theorem.

Theorem B.1 (COLI-Ordering of OOH Measures). *Let $p^{h,t}_{COLI}$ denote the COLI-consistent price of one period's housing services in period t. Under the maintained assumptions of competitive long-run equilibrium, geometric depreciation, and positive expected capital gains:*

$$\underbrace{p^{h,t}_{pay}}_{Payments} \le \underbrace{p^{h,t}_{COLI}}_{True\ COLI} = U_t = R_t^{new,adj} \le R_t^{new} \le R_t^{contract} \tag{B.18}$$

where $R_t^{new,adj}$ is the new-contract rent adjusted for landlord-specific costs via equation (B.11), and the acquisitions price P_t^H is not directly comparable to the flow measures above because it indexes an asset stock, not a period service flow.

The theorem implies:

- The **user cost** U_t and the **adjusted new-contract rent** $R_t^{\text{new,adj}}$ are theoretically equivalent in equilibrium and both provide COLI-consistent measures of OOH services.

- The **unadjusted new-contract rent** R_t^{new} overstates the COLI price by the landlord friction wedge $(\delta^R - \delta^O)(1 + \pi^e)s_S + m_t + \nu_t + \pi_t^{\text{ins}} \geq 0$.

- The **contract (roll-over) rent** R_t^{contract} further overstates the COLI price by the stickiness wedge $\sigma_t = R_t^{\text{new}} - R_t^{\text{contract}} \geq 0$ during appreciation episodes.

- The **payments approach** understates the COLI price because it ignores the equity opportunity cost and the capital gain offset.

- The **acquisitions approach** is not a measure of the COLI period price at all: it measures asset price inflation ($\approx \pi_t^e$), not the service flow cost ($\approx r_t^* + \delta$). The two coincide only if $r_t^* + \delta = \pi_t^e$, an implausible condition.

These results motivate the empirical strategy of the main paper: to compare all three approaches in the Tokyo data and to quantify the gaps in (B.18) over different market episodes. The largest gaps are predicted to occur during episodes of rapid asset price appreciation (large positive π_t^e, large positive stickiness wedge σ_t), exactly the configuration observed during the Tokyo bubble of 1987–1990 and the post-COVID appreciation of 2021–2025.

C Depreciation Models:

General Framework, Straight Line, and One Hoss Shay

In this section, a "general" model of depreciation for durable goods that appear on the market each period without undergoing quality change will be presented. In the following three sections, this general model will be specialized to the three most common models of depreciation that appear in the literature. Additional problems that arise when the durable is a unique good (or when second hand markets do not exist) are discussed in the following section of this appendix.

The main tool that can be used to identify depreciation rates for a durable good is the cross sectional sequence of asset prices classified by their age that units of the good sell for on the second hand market at any point of time.[13] Thus in order to apply this method of measurement, it is necessary that such second hand markets exist.

Some notation is required. Let P_0^t be the price of a newly produced unit of the durable good at the *beginning* of period t. Let P_v^t be the second hand market price at the beginning of period t of a unit of the durable good that is v periods old.[14] The *beginning of period t cross sectional depreciation rate* for a brand new unit of the durable good, δ_0^t, is defined as follows:

$$1 - \delta_0^t \equiv P_1^t / P_0^t. \tag{C.1}$$

Once δ_0^t has been defined by (C.1), the *period t cross sectional depreciation rate* for a unit

[13] Another information source that could be used to identify depreciation rates for the durable good is the sequence of vintage rental or leasing prices that might exist for some consumer durables. In the closely related capital measurement literature, the general framework for an internally consistent treatment of capital services and capital stocks in a set of vintage accounts was set out by Jorgenson (1989) and Hulten 1990, pp. 127–129 1996, pp. 152–160.

[14] If these second hand vintage prices depend on how intensively the durable good has been used in previous periods, then it will be necessary to further classify the durable good not only by its vintage v but also according to the intensity of its use. In this case, think of the sequence of vintage asset prices P_v^t as corresponding to the prevailing market prices of the various vintages of the good at the beginning of period t for assets that have been used at "average" intensities.

of the durable good that is one period old at the beginning of period t, δ_1^t, can be defined using the following equation:

$$(1-\delta_1^t)(1-\delta_0^t) \equiv P_2^t/P_0^t. \tag{C.2}$$

Note that P_2^t is the beginning of period t asset price of a unit of the durable good that is 2 periods old and it is compared to the price of a brand new unit of the durable, P_0^t.

Given that the period t cross sectional depreciation rates for units of the durable that are $0, 1, 2, \ldots, v-1$ periods old at the beginning of period 0 are defined (these are the depreciation rates $\delta_0^t, \delta_1^t, \delta_2^t, \ldots, \delta_{v-1}^t$), then the *period t cross sectional depreciation rate for units of the durable that are v periods old* at the beginning of period t, δ_v^t, can be defined using the following equation:

$$(1-\delta_v^t)(1-\delta_{v-1}^t)\cdots(1-\delta_1^t)(1-\delta_0^t) \equiv P_{v+1}^t/P_0^t. \tag{C.3}$$

Thus it is clear how the sequence of *period 0 vintage asset prices* P_v^t can be converted into a sequence of *period t vintage depreciation rates*, δ_v^t. In the depreciation literature, it is usually assumed that the sequence of vintage depreciation rates, δ_v^t, is independent of the period t so that:

$$\delta_v^t = \delta_v \qquad \text{for all periods } t \text{ and all ages v.} \tag{C.4}$$

The above material shows how the sequence of vintage or used durable goods prices at a point in time can be used in order to estimate depreciation rates. This method for estimating depreciation rates using data on second hand assets, with a few extra modifications to account for differing ages of retirement, was pioneered by Beidelman (1973, 1976) and Hulten and Wykoff (1981a, 1981b, 1996).[15]

[15] See also Jorgenson (1996) for a review of the empirical literature on the estimation of depreciation rates.

Recall the user cost formula for a new unit of the durable good under consideration which was defined above by the end-of-period user cost expression. The same approach can be used in order to define a sequence of period 0 user costs for all vintages v of the durable. Thus suppose that P_{v+1}^{1a} is the *anticipated end of period 0 price* of a unit of the durable good that is v periods old at the beginning of period 0 and let r^0 be the consumer's opportunity cost of capital for period 0. Then the discounted to the beginning of period 0 *user cost* of a unit of the durable good that is v periods old at the beginning of period 0, u_v^0, is defined as follows:

$$u_v^0 \equiv P_v^0 - P_{v+1}^{1a}/(1 + r^0); \qquad v = 0, 1, 2, \ldots \tag{C.5}$$

It is now necessary to specify how the *end* of period 0 anticipated vintage asset prices P_v^{1a} are related to their counterpart *beginning* of period 0 vintage asset prices P_v^0. The assumption that is made now is that the entire sequence of vintage asset prices at the end of period 0 is equal to the corresponding sequence of asset prices at the beginning of period 0 times a general anticipated period 0 inflation rate factor, $(1 + i^0)$, where i^0 is the *anticipated period 0 (general) asset inflation rate*. Thus it is assumed that[16]

$$P_v^{1a} = (1 + i^0)P_v^0; \qquad v = 0, 1, 2, \ldots \tag{C.6}$$

Substituting (C.6) and (C.1)-(C.4) into (C.5) leads to the following beginning of *period 0*

[16] More generally, we assume that assumptions (C.6) hold for subsequent periods t as well; i.e., it is assumed that $P_v^{t+1a} = (1 + i^t)P_v^t$ for $v = 0, 1, 2, \ldots$ and $t = 0, 1, 2, \ldots$ where P_v^{t+1a} is the anticipated price of a unit of the durable good that is v periods old at the end of period t, i^t is a period t expected asset inflation rate for all ages of the durable and P_v^t is the second hand market price for a unit of the durable good that is v periods old at the beginning of period t.

sequence of vintage user costs:[17]

$$u_v^0 = (1-\delta_{v-1})(1-\delta_{v-2})\cdots(1-\delta_0)[(1+r^0)-(1-\delta_v)(1+i^0)]P_0^0/(1+r^0)$$
$$= (1-\delta_{v-1})(1-\delta_{v-2})\cdots(1-\delta_0)[r^0-i^0+\delta_v(1+i^0)]\frac{P_0^0}{1+r^0}; \quad v=1,2,\ldots \qquad \text{(C.7)}$$

If $v=0$, then $u_0^0 \equiv [r^0-i^0+\delta_0(1+i^0)]P_0^0/(1+r^0)$ and this agrees with the user cost formula for a new purchase of the durable u^0 that was derived earlier (with our changes in notation; i.e., P^0 is now called P_0^0).

The sequence of vintage user costs u_v^0 defined by (C.7) are expressed in terms of prices that are discounted to the *beginning* of period 0. However, as was done in Appendix A above, it is also possible to express the user costs in terms of prices that are "anti-discounted" to the *end* of period 0. Thus define the sequence of vintage *end of period 0 user cost* p_v^0 as follows:

$$p_v^0 \equiv (1+r^0)u_v^0 = (1-\delta_{v-1})(1-\delta_{v-2})\cdots(1-\delta_0)$$
$$\times [r^0-i^0+\delta_v(1+i^0)]P_0^0; \quad v=1,2,\ldots. \qquad \text{(C.8)}$$

with p_0^0 defined as follows:

$$p_0^0 \equiv (1+r^0)u_0^0 = [r^0-i^0+\delta_v(1+i^0)]P_0^0. \qquad \text{(C.9)}$$

Thus if the price statistician has estimates for the vintage depreciation rates δ_v and the real interest rate r^{0*} and is able to collect a sample of prices for new units of the durable good P_0^0, then the sequence of vintage user costs defined by (C.8) can be calculated. To complete the model, the price statistician should gather information on the stocks held by the household sector of each vintage of the durable good and then normal index number theory can be

[17]When $v=0$, define $\delta_{-1} \equiv 1$; i.e., the terms in front of the square brackets on the right hand side of (C.7) are set equal to 1.

applied to these p's and q's, with the p's being vintage user costs and the q's being the vintage stocks pertaining to each period. For some worked examples of this methodology under various assumptions about depreciation rates and the calculation of expected asset inflation rates, see Diewert and Lawrence (2000) and Diewert (2005a).[18]

In the following three sections, the general methodology described above is specialized by making additional assumptions about the form of the vintage depreciation rates δ_v.[19]

C.1 Straight Line Depreciation

Another very common model of depreciation is the *straight line model.*[20] In this model, a most probable length of life for the durable is somehow determined, say L periods, so that after being used for L periods, the durable is scrapped. In the straight line depreciation model, it is assumed that the period 0 cross sectional vintage asset prices P_v^0 decline in a linear fashion relative to the period 0 price of a new asset P_0^0:

$$P_v^0/P_0^0 = [L - v]/L \qquad \text{for } v = 0, 1, 2, \ldots, L-1. \tag{C.10}$$

For $v = L, L+1, \ldots$, it is assumed that $P_v^0 = 0$. Now use definitions (C.5) and (C.8) along with assumptions (C.6) in order to obtain the following sequence of *end of period 0 vintage*

[18] Additional examples and discussion can be found in two recent OECD Manuals on productivity measurement and the measurement of capital; see Schreyer (2001, 2009).

[19] In the case of one hoss shay depreciation, assumptions are made about the sequence of user costs, u_v^t, as the age v varies.

[20] This model of depreciation dates back to the late 1800's; see Matheson (1910, p. 55), Garcke and Fells (1893, p. 98) or Canning (1929, pp. 265–266).

user costs for a unit of the durable good of age v at the beginning of period 0:

$$\begin{aligned} p_v^0 &= P_v^0(1+r^0) - (1+i^0)P_{v+1}^0 \qquad \text{for } v = 0,1,2,\ldots,L-1 \\ &= \tfrac{1}{L}[(L-v)(1+r^0) - (L-v-1)(1+i^0)]P_0^0 \\ &\quad \text{(using assumptions (C.10))} \\ &= \left[(r^0 - i^0)(L-v)L^{-1} + (1+i^0)L^{-1}\right]P_0^0. \end{aligned} \tag{C.11}$$

The user costs for units of the durable good that are older than L periods are zero; i.e., $p_v^0 \equiv 0$ for $v \geq L$. Looking at the terms in square brackets on the right hand side of (C.11), it can be seen that the first term $(r^0 - i^0)(L-v)P_0^0/L$ is a real interest opportunity cost for holding and using the unit of the durable that is v periods old (and this imputed real interest cost declines as the durable good ages; i.e., as the age v increases) and the second term $(1+i^0)(1/L)P_0^0$ is an inflation adjusted depreciation term that is equal to the constant straight line depreciation rate $1/L$ times the adjustment factor for asset inflation over the period, $(1+i^0)$, times the price of a new unit of the durable good P_0^0. Note that in period t, the corresponding end of period user cost for a unit of the durable good that is v periods old is $p_v^t \equiv [(r^t - i^t)(L-v)L^{-1} + (1+i^t)L^{-1}]P_0^t$ for $v = 0,1,2,\ldots,L-1$. Thus in both periods 0 and t, the sequences of end of period user costs by age, $\{p_v^0\}$ and $\{p_v^t\}$ for $v = 0,1,2,\ldots,L-1$, are proportional to the price of a new unit of the durable for periods 0 and t, P_0^0 and P_0^t respectively[21] but if r^0 and/or i^0 change to a different r^t or i^t, then the factors of proportionality will change as we go from period 0 to t and so we cannot apply Hicks' Aggregation Theorem in this case. Thus in the case of changing nominal interest rates r and/or changing expected or actual asset price inflation rates, i^t, we cannot assume that the overall inflation rate between periods 0 and t for all ages of the durable good is equal to

[21] Thus as the price of a new unit of the durable good changes over time, the value of depreciation will also change in line with the change in the price of the new unit. Thus economic depreciation as we have defined it is different from historical cost accounting depreciation which does not adjust depreciation allowances for changes in the levels of asset prices over time.

P_0^t/P_0^0 as was the case with the geometric model of depreciation. Thus for the straight line model of depreciation, it is necessary to keep track of household purchases of the durable for L periods and weight up each vintage quantity q^{-v} of these purchases by the corresponding end of period user costs vintage user cost p_v^0 defined by (C.11) for period 0 and a similar calculation of household holdings of the durable good by age for period t along with the period t counterparts to the period 0 user costs defined by (C.11) will be necessary. Once these vectors of prices and quantities have been calculated for both periods, then normal index number theory can be applied to get the overall price index for the household holdings of the durable good and this index can be used to deflate the user cost aggregate values to get an appropriate volume index.[22] Thus the straight line model of depreciation is considerably more complicated to implement than the geometric model of depreciation explained in the previous section.[23]

C.2 One Hoss Shay or Light Bulb Depreciation

The final model of depreciation that is in common use is the "light bulb" or *one hoss shay model of depreciation*.[24] In this model, the durable delivers the *same* services for each vintage:

[22]Diewert and Lawrence (2000) noted this problem with the straight line model of depreciation; i.e., that in general, *an index number formula* should be used to aggregate over the different ages of the asset in order to obtain an aggregate of the capital services of the different vintages of the asset.

[23]However, if one is willing to assume that the reference interest rate for period t, r^t, and the expected asset inflation rate over all ages of the asset, i^t, both remain constant, then all reasonable index number formula will estimate the overall rate of user cost inflation between periods 0 and t as the new good price ratio, P_0^t/P_0^0. However, the assumption that r^t and i^t remain constant over time is only a rough approximation to reality. Note that in order to calculate real and nominal consumption of the durable (over all ages of the durable), it will be necessary to use the vintage user costs defined by (C.11) for a constant r and i to weight up past purchases of the durable good. Thus define the constants $\alpha_v \equiv [(r-i)(L-v)L^{-1}+(1+i)L^{-1}]$ for $v = 0, 1, 2, \ldots, L-1$ and $\alpha_v \equiv 0$ for $v \geq L$. Then the period t *nominal value of durable services* is defined as $v^t \equiv p_0^t q^t + p_1^t q^{t-1} + p_2^t q^{t-2} + \cdots + p_{L-1}^t q^{t-L+1} = \alpha_0 P_0^t q^t + \alpha_1 P_0^t q^{t-1} + \alpha_2 P_0^t q^{t-2} + \cdots + \alpha_{L-1} P_0^t q^{t-L+1} = P_0^t Q^t$ where Q^t is the real value or volume of durable services defined as $Q^t \equiv \alpha_0 q^t + \alpha_1 q^{t-1} + \alpha_2 q^{t-2} + \cdots + \alpha_{L-1} q^{t-L+1}$. Define $\beta_v \equiv (L-v)/L$ for $v = 0, 1, 2, \ldots, L-1$. The period t *asset value* of consumer holdings of the durable good is defined as $V^t \equiv P_0^t q^t + P_1^t q^{t-1} + P_2^t q^{t-2} + \cdots + P_{L-1}^t q^{t-L+1} = P_0^t[\beta_0 q^t + \beta_1 q^{t-1} + \beta_2 q^{t-2} + \cdots + \beta_{L-1} q^{t-L+1}] = P_0^t Q^{t*}$ where we have used assumptions (C.10) applied to period t and the real value of durable stocks held by households at the end of period t is defined as $Q^{t*} \equiv \beta_0 q^t + \beta_1 q^{t-1} + \beta_2 q^{t-2} + \cdots + \beta_{L-1} q^{t-L+1}$. The decomposition of V^t into $P_0^t Q^{t*}$ does not require the assumption of constant r^t and i^t.

[24]This model can be traced back to Böhm-Bawerk (1891); 342. For a more comprehensive exposition, see Hulten (1990); 124 or Diewert (2005a).

a chair is a chair, no matter what its age is (until it falls to pieces and is scrapped). Thus this model also requires an estimate of the most probable life L of the consumer durable.[25] In this model, it is assumed that the sequence of vintage beginning of the period user costs u_v^0 defined by (C.5) and (C.6) is *constant* for all vintages younger than the asset lifetime L; i.e., it is assumed that

$$u_v^0 \equiv P_v^0 - (1+i^0)P_{v+1}^0/(1+r^0) = u^0; \qquad v = 0, 1, 2, \ldots, L-1, \tag{C.12}$$

where $u^0 > 0$ is a constant. Equations (C.12) can be rewritten in the following form:

$$u^0 = P_v^0 - \gamma P_{v+1}^0; \qquad v = 0, 1, 2, \ldots, L-1, \tag{C.13}$$

where the *discount factor* γ is defined as

$$\gamma \equiv (1+i^0)/(1+r^0) = 1/(1+r^{0*}). \tag{C.14}$$

The interest rate r^{0*} can be regarded as an *asset specific real interest rate*; i.e., $1 + r^{0*} \equiv (1+r^0)/(1+i^0)$ so that one plus the nominal interest rate r^0 is deflated by one plus the expected asset price inflation rate, i^0. Note that equations (C.13) can be rewritten as follows:

$$P_v^0 = u^0 + \gamma P_{v+1}^0; \qquad v = 0, 1, 2, \ldots, L-1. \tag{C.15}$$

Use equation (C.15) with $v = 0$ to express P_0^0 in terms of u^0 and P_1^0. Now use (C.15) with $v = 1$ to express P_2^0 in terms of u^0 and P_1^0 and then substitute out P_1^0 using the previous

[25] The assumption of a single life L for a durable can be relaxed using a methodology due to Hulten: "We have thus far taken the date of retirement T to be the same for all assets in a given cohort (all assets put in place in a given year). However, there is no reason for this to be true, and the theory is readily extended to allow for different retirement dates. A given cohort can be broken into components, or subcohorts, according to date of retirement and a separate T assigned to each. Each subcohort can then be characterized by its own efficiency sequence, which depends among other things on the subcohort's useful life T_i." Charles R. Hulten 1990, p. 125.

expression that expressed P_1^0 in terms of P_0^0 and u^0. Continue this substitution process until finally it ends after L such substitutions when P_L^0 is reached and of course, P_L^0 equals zero. The following equation is obtained:

$$\begin{aligned}
P_0^0 &= u^0 + \gamma u^0 + \gamma^2 u^0 + \cdots + \gamma^{L-1} u^0 \\
&= u^0[1 + \gamma + \gamma^2 + \cdots + \gamma^{L-1}] \\
&= \{u^0/(1-\gamma)\} - \{u^0 \gamma^L/(1-\gamma)\} \qquad \text{provided that } \gamma < 1^{26} \\
&= u^0(1-\gamma^L)/(1-\gamma). \qquad \text{(C.16)}
\end{aligned}$$

Now use the last equation in (C.16) in order to solve for the constant over vintages (beginning of the period) *user cost* for this model, u^0, in terms of the period 0 price for a new unit of the durable, P_0^0, and the discount factor γ defined by (C.15):

$$u^0 = (1-\gamma)P_0^0/(1-\gamma^L) = u_v^0; \qquad v = 0, 1, 2, \ldots, L-1. \qquad \text{(C.17)}$$

The sequence of *end of period 0 user cost*, p_v^0, is as usual, equal to the corresponding beginning of the period 0 user cost, u_v^0, times the period 0 nominal interest rate factor, $1 + r^0$:

$$p_v^0 \equiv (1+r^0)u_v^0 = [1+r^0][1-\gamma^0][1-(\gamma^0)^L]^{-1} P_0^0 = p_0^0; \qquad v = 0, 1, 2, \ldots, L-1, \quad \text{(C.18)}$$

and $p_v^0 = 0$ for $v = L, L+1, \ldots$ and $\gamma^0 \equiv (1+i^0)/(1+r^0)$.

The *aggregate services of all vintages* of the good for period 0, including those purchased in

[26] If $\gamma \geq 1$, then use the second equation in (C.16) to express u^0 in terms of P_0^0 and the various powers of γ.

period 0, will have the following value, v^0:

$$\begin{aligned} v^0 &= p_0^0 q^0 + p_1^0 q^{-1} + p_2^0 q^{-2} + \cdots + p_{L-1}^0 q^{-(L-1)} \\ &= p_0^0 [q^0 + q^{-1} + q^{-2} + \cdots + q^{-(L-1)}] \\ &= p_0^0 Q^0, \end{aligned} \tag{C.19}$$

where the *period 0 aggregate (quality adjusted) quantity of durable services* consumed in period 0, Q^0, is defined as follows for this depreciation model:

$$Q^0 \equiv q^0 + q^{-1} + q^{-2} + \cdots + q^{-(L-1)}. \tag{C.20}$$

Thus in this model of depreciation, the service quantity aggregate is the simple sum of household purchases over the last L periods.[27] As was the case with the geometric depreciation model, the one hoss shay model does not require index number aggregation over vintages when calculating aggregate services from all vintages of the durable: there is a constant service price p_0^0 for all assets that are less than L periods old and the associated period 0 quantity Q^0 is the simple sum defined by (C.20) over the purchases of the last L periods for the one hoss shay model.[28]

The first two models of depreciation considered in this appendix (geometric and straight-line depreciation) made assumptions about the pattern of depreciation rates for durables of different ages. The model in this section made assumptions about the pattern of user costs for durable goods of different ages. For a more general model of depreciation that allows for

[27] In the national income accounting literature, this measure is sometimes called the gross capital stock.

[28] Using equations (C.15), it can be shown that $P_v^0 = u^0[1 + (\gamma^0) + (\gamma^0)^2 + \cdots + (\gamma^0)^{L-1-v}]$ for $v = 0, 1, 2, \ldots, L-1$ where $\gamma^0 \equiv (1+i^0)/(1+r^0)$ and $P_v^0 = 0$ for $v \geq L$. Thus the period 0 value of the stock of consumer durables is $\sum_{v=0}^{L-1} P_v^0 q^{-v}$. The corresponding asset prices for period t are equal to $P_v^t = u^t[1 + (\gamma^t) + (\gamma^t)^2 + \cdots + (\gamma^t)^{L-1-v}]$ for $v = 0, 1, 2, \ldots, L-1$ where $u^t \equiv [1-(\gamma^t)]P_0^t/[1-(\gamma^t)^L]$, $\gamma^t \equiv (1+i^t)/(1+r^t)$ and $P_v^t = 0$ for $v \geq L$. The period t value of the stock of consumer durables is $\sum_{v=0}^{L-1} P_v^t q^{t-v}$. An index number formula will have to be used to form aggregate price and quantity indexes for the *stocks* of consumer durables using the one hoss shay model of depreciation.

an arbitrary pattern of user costs by age of asset, see Diewert and Wei (2017).

How can the different models of depreciation be distinguished empirically? For durable goods that do not change in quality over time, there are *three possible methods* for determining the sequence of vintage depreciation rates:[29]

- By making a rough estimate of the average length of life L for the durable good and then by *assuming* a depreciation model that seems most appropriate.[30]
- By using cross sectional information on used durable prices at a single point in time and then using equations (C.1)-(C.3) above to determine the corresponding sequence of vintage depreciation rates.[31]
- By using cross sectional information on the rental or leasing prices of the durable as a function of the age of the durable and then equations (C.8) and (C.9), along with information on the appropriate nominal interest rate r^0 and expected durables inflation rate i^0 along with information on the price of a new unit of the durable good P^0 can be used to determine the corresponding sequence of vintage depreciation rates.

C.3 Estimating Depreciation Rates from Hedonic Regressions

The three depreciation models described in this appendix (Sections C.1–C.3) —geometric, straight-line, and one hoss shay—each require the depreciation rate δ (or the asset lifetime L) as a parameter input. For non-housing consumer durables, this parameter is often calibrated from engineering data or national accounts benchmarks. For housing, however, a more rigorous approach is available: the depreciation rate can be estimated *directly from*

[29] These three classes of methods were noted in Malpezzi, Ozanne and Thibodeau (1987, pp. 373–375) in the housing context.

[30] A length of life L is can be converted into an equivalent geometric depreciation rate δ by setting δ equal to a number between $1/L$ and $2/L$.

[31] This method will be pursued in sections 12 and 13 for housing depreciation rates.

transaction data using hedonic regression methods. This section introduces the two principal hedonic approaches to depreciation estimation and explains how they connect to the full hedonic framework developed in Appendix D.

The Identification Problem

The central challenge in estimating housing depreciation is that the structure component of a dwelling's value—which depreciates—cannot be directly observed in a transaction price. What is observed is the total selling price V_{tn}, which reflects the combined value of the structure *and* the land:

$$V_{tn} \;=\; \underbrace{P_{Lt}\,L_{tn}}_{\text{land value}} \;+\; \underbrace{P_{St}(1-\delta)^{A_{tn}}\,S_{tn}}_{\text{structure value}} + \varepsilon_{tn} \tag{C.21}$$

where P_{Lt} is the period-t land price per square metre, L_{tn} is the land area, P_{St} is the period-t construction cost per square metre, δ is the geometric depreciation rate, A_{tn} is the building age, and S_{tn} is the floor space area. This is the *builder's model* of Appendix D (equations (D.2)–(D.3)).

Two approaches have been developed to extract δ from this model.

Method 1: The Cross-Sectional Age–Price Profile (Builder's Model)

The first approach embeds δ as a parameter to be estimated directly within the builder's hedonic regression (C.21). Because the land price P_{Lt} and the construction cost P_{St} vary over time, the nonlinear regression is typically estimated jointly over all time periods, with the structure price index p_{St} (the MLIT construction cost index) supplied exogenously to resolve the collinearity between L_{tn} and S_{tn}. The estimating equation becomes

$$V_{tn} \;=\; \alpha_t\Big(\textstyle\sum_{j=1}^{J}\omega_j D_{W,tn,j}\Big) f_L(L_{tn}) \;+\; p_{St}(1-\delta)^{A_{tn}}\, g_S(S_{tn}) \;+\; \varepsilon_{tn} \tag{C.22}$$

where α_t are land price time-dummy coefficients, ω_j are ward-level relative land price parameters, $f_L(\cdot)$ is a spline function adjusting for lot size, and $g_S(\cdot)$ is a spline function adjusting for floor space; see Appendix D, equations (D.9)–(D.13). The depreciation rate δ is estimated jointly with the other parameters by nonlinear least squares.

Properties of the estimate. The estimate of δ obtained from (C.22) is a *net* depreciation rate: it reflects gross physical deterioration of the structure less any quality improvements due to renovation and maintenance, since neither is separately observable in transaction data.[32] The method can accommodate a non-constant (piecewise-linear) depreciation schedule by replacing $(1-\delta)^{A_{tn}}$ with a spline function of building age, as in Diewert and Shimizu (2015) (equations (12)–(13) of that paper). Their preferred specification yields the following estimates for Tokyo single-family houses:

$$\begin{aligned} \hat{\delta}_1 &= 2.47\% \text{ (age 0–10 yr)}, \\ \hat{\delta}_2 &= 1.59\% \text{ (age 10–20 yr)}, \\ \hat{\delta}_3 &= 0.32\% \text{ (age 20–50 yr)} \end{aligned} \tag{C.23}$$

These estimates reveal that new structures depreciate rapidly but that surviving older structures exhibit near-zero net depreciation, consistent with extensive renovation of long-lived buildings. For Tokyo condominiums, Diewert and Shimizu (2016) apply the geometric specification and obtain $\hat{\delta} \approx 3.6\%$ per year—higher than for single-family houses, reflecting the faster physical deterioration of reinforced concrete high-rise structures.

[32] This point is noted explicitly in Diewert and Shimizu (2015, p. 1663, footnote 10). The net depreciation rate is the appropriate concept for the owner-occupier user cost, since renovation expenditure is separately covered in the CPI basket.

Method 2: The Log-Linear Age Coefficient (Time-Dummy Hedonic)

The second approach uses the simpler log-linear time-dummy hedonic regression, in which the logarithm of the price per square metre is regressed on building age as a continuous variable:

$$\ln(V_{tn}/S_{tn}) \;=\; \alpha_0 + \beta_{\text{age}}\, A_{tn} + \sum_t \tau_t\, \mathbf{1}(\text{period} = t) + \boldsymbol{\beta}'\mathbf{x}_{tn} + \varepsilon_{tn} \tag{C.24}$$

where A_{tn} is building age in months and $\mathbf{x}_{tn}$ collects other quality characteristics (transit access, location dummies, building structure). This is the specification used in Appendix F (equations (A.1) and (A.2)).

The relationship between the estimated coefficient $\hat{\beta}_{\text{age}}$ (per month) and the annual depreciation rate is:

$$\hat{\delta}_{\text{annual}} \;=\; -\hat{\beta}_{\text{age}} \times 12 \tag{C.25}$$

(taking the sign convention that $\hat{\beta}_{\text{age}} < 0$). Applied to the rolling-window hedonic in Appendix F (Table F.9), this yields $\hat{\delta} \approx 0.186 \times 12 = 2.23\%$ per year (pooled) and 2.18% on average across rolling windows (Table F.10).

Comparison of the two methods. The log-linear age coefficient method (Method 2) is computationally simpler and can be estimated by OLS, but it imposes a *constant* geometric depreciation rate over all building ages, which may be too restrictive. The builder's model method (Method 1) is nonlinear but allows for a flexible, age-varying depreciation schedule and provides a structural decomposition of value into land and structure components. Both methods yield net depreciation rates that are consistent with the calibration $\delta = 0.020$ (2.0% per year) adopted in this paper; see the detailed comparison in Appendix F, Section F.8.

Connection to Appendix D

The full development of both hedonic methods—including the identification strategy for the builder's model, the treatment of land–structure collinearity, the spline specifications for lot size and floor space, and the rolling-window implementation for the log-linear approach—is presented in Appendix D. In particular:

- The **builder's model** (Method 1) is derived as equations (D.1)–(D.15) in Appendix D, progressing from the basic model to the full nonlinear specification with location controls and flexible functional forms.
- The **log-linear model** (Method 2) corresponds to the time-dummy hedonic in Appendix D, Section D, and is applied to the full Tokyo panel in Appendix F, Sections F.2–F.3.

The reader is referred to Appendix D for the technical details of the estimation procedure, the identification of the land and structure components of property value, and the construction of constant-quality price indexes from the estimated hedonic surface.

Empirical estimates for Tokyo. Table C.7 summarises the geometric depreciation rate estimates obtained by applying both methods to the Tokyo condominium transaction data (Appendix F, Sections F.2–F.3). The pooled hedonic estimate yields $\hat{\delta} = 0.0204$ per annum (SE $= 0.0003$), and the rolling-window mean is 0.0203 (SD across 467 windows $= 0.0006$). Both estimates are mutually consistent and align closely with the MLIT benchmark of $\delta = 0.020$. This convergence across estimation methods validates the calibration $\delta = 0.020$ adopted throughout the user cost calculations in Appendix E and the main text.

Table C.7: Summary of Structure Depreciation Rate Estimates for Tokyo, 1986–2025

Method	$\hat{\delta}$	SE	95% CI	N
Age coefficient (pooled hedonic)	0.0204	0.0003	[0.0198, 0.0210]	357,627
Age coefficient (rolling, mean)	0.0203	0.0006	[0.0191, 0.0215]	467 windows
Benchmark (MLIT/literature)	0.020	—	—	—
Calibration used in this paper	$\delta = 0.020$ (rounded to 3 s.f.)			

$\hat{\delta}$ is the annualised geometric depreciation rate estimated from the building-age coefficient in the hedonic regression: $\hat{\delta} = -\hat{\beta}_A \times 12$. Pooled hedonic: Appendix F, Table F.9; rolling windows: Table F.10. Both estimates lie within 0.001 of the MLIT benchmark, supporting $\delta = 0.020$ as the calibration throughout this paper.

D Hedonic Regression Models for Housing: Land–Structure Decomposition and Demand-Side Models

In this section, the problems associated with the construction of constant quality residential property price indexes will be studied. In this section, we will look at the construction of constant quality indexes for the *stock of residential housing units*; in subsequent sections, we will look at the problems associated with pricing the services of a residential dwelling unit.

There are two difficult measurement problems associated with the construction of a constant quality house price index:

- A dwelling unit is a *unique consumer durable good*; i.e., the location of a housing unit is a price determining characteristic of the unit and each house or apartment has a unique location.
- There are two main components of a dwelling unit: (i) *the size of the structure* (measured in square meters of floor space) and (ii) *the size of the land plot* that the structure sits on (also measured in square meters). However, the purchase price of a dwelling unit is for the entire property and thus the decomposition of property price into its two main components will involve imputations.

The first problem area listed above might not be a problem if the same dwelling unit sold at market prices at a frequent rate so that the location would be held constant and it would seem that the usual matched model methodology that is used in constructing price indexes could be applied. But houses do not transact all that frequently; typically, a house is held for 10-20 years by the same owner before it is resold. Moreover, the structure is not constant over time; depreciation of the structure occurs over time and owners renovate and replace aging components of the structure. For example, the roofing materials for many dwellings are replaced every 20 or 30 years. Thus depreciation and renovation constantly change the quality of the structure.

The second problem area is associated with the difficulty of decomposing the transaction price for a housing unit into *separate components* representing the structure value and the land value; i.e., the single property price is for both components of the housing unit but for many purposes, we require separate valuations for the two components. The international System of National Accounts, requires separate valuations for the land and structure components of residential housing in the National Balance Sheets of the country. Many countries construct estimates for the Total Factor Productivity or Multifactor Productivity of the various sectors in the economy and the methodology used to construct these estimates requires separate price and quantity information on both structures and the land that the structures sit on. In this section, we will indicate a possible method that can be used to accomplish this decomposition of property value into constant quality land and structure components.

The *builder's model* for valuing a detached dwelling unit postulates that the value of the property is the sum of two components: the value of the land which the structure sits on plus the value of the structure. This model can be justified in two situations:

- A household purchases a residential land plot with no structure on it (or if there are structures on the land plot, they are immediately demolished).[33]

[33]The cost of the demolition should be added to the purchase price for the land to get the overall land

- A household purchases a land plot and immediately builds a new dwelling unit on the property.

In the first case, it is clear that the property value is equal to the land value. In the second case, The total cost of the property after the structure is completed will be equal to the floor space area of the structure, say S square meters, times the building cost per square meter β_t during period t, plus the cost of the land, which will be equal to the cost per square meter α_t times the area of the land site, say L square meters. Now think of a sample of properties of the same general type in the same general location, which have prices or values V_{tn} in period t (where $t = 1, \ldots, T$) and structure floor space areas S_{tn} and land areas L_{tn} for $n = 1, \ldots, N(t)$ where $N(t)$ is the number of observations in period t. Assume that these prices are equal to the sum of the land and structure costs plus error terms ε_{tn} which we assume are independently normally distributed with zero means and constant variances. This leads to the following *hedonic regression model* for period t where the α_t and β_t are the parameters to be estimated in the regression:[34]

$$V_{tn} = \alpha_t L_{tn} + \beta_t S_{tn} + \varepsilon_{tn}; \qquad t = 1, \ldots, T; n = 1, \ldots, N(t). \tag{D.1}$$

The hedonic regression model defined by (D.1) applies to new structures and to purchases of vacant residential lots in the neighbourhood under consideration where $S_{tn} = 0$. Note that there are some strong simplifying assumptions built into the model defined by (D.1): (i) the period t land price α_t (per m^2) is assumed to be constant across all properties in the neighbourhood under consideration and (ii) the construction cost (per m^2) is also assumed to be constant across all housing units built in the neighbourhood during period t. The above

price for the land plot.

[34] Other papers that have suggested hedonic regression models that lead to additive decompositions of property values into land and structure components include Clapp (1980, pp. 257–258), Bostic, Longhofer and Redfearn (2007, p. 184), Francke and Vos (2004), Diewert (2008, pp. 19–22; 2010), Francke (2008, p. 167), Koev and Santos Silva (2008), Rambaldi, McAllister, Collins and Fletcher (2010), Diewert, de Haan and Hendriks (2011, 2015), Eurostat (2013), Diewert and Shimizu (2015, 2016, 2017a), Burnett-Isaacs, Huang and Diewert (2016), and Diewert, Huang and Burnett-Isaacs (2017).

model applies to raw land purchases and the purchases of new dwelling units during period t in the neighbourhood under consideration. It is likely that a model that is similar to (D.1) applies to sales of older structures as well. Older structures will be worth less than newer structures due to the *depreciation* of the structure. Assuming that we have information on the age of the structure n at time t, say $A(t,n)$, and assuming a geometric (or declining balance) depreciation model, a more realistic hedonic regression model than that defined by (D.1) above is the following *basic builder's model*:

$$V_{tn} = \alpha_t L_{tn} + \beta_t (1-\delta)^{A(t,n)} S_{tn} + \varepsilon_{tn}; \qquad t = 1, \ldots, T; n = 1, \ldots, N(t), \tag{D.2}$$

where the parameter δ reflects the *net geometric depreciation rate* as the structure ages one additional period. Thus if the age of the structure is measured in years, we would expect an annual *net* depreciation rate to be around 1 to 3 per cent per year.[35] Note that (D.2) is now a nonlinear regression model whereas (D.1) was a simple linear regression model. The period t constant quality price of land will be the estimated coefficient for the parameter α_t and the price of a unit of a newly built structure for period t will be the estimate for β_t. The period t quantity of land for property n is L_{tn} and the period t quantity of structure for property n, expressed in equivalent units of a new structure, is $(1-\delta)^{A(t,n)} S_{tn}$ where S_{tn} is the floor space area of the structure for property n in period t.

Note that the above model can be viewed as a *supply side model* as opposed to a *demand side model*.[36] Basically, we are assuming a valuation of a housing structures that is equal to the cost per unit floor space area of a new unit times the floor space area times an adjustment for structure depreciation. The corresponding land value of the property is determined residually as total property value minus the imputed value of structures quality adjusted for the age

[35] This estimate of depreciation is regarded as a *net depreciation rate* because it is equal to a "true" gross structure depreciation rate less an average renovations appreciation rate. Since typically information on renovations and major repairs to a structure is not available, the age variable will only pick up average gross depreciation less average real renovation expenditures.

[36] We will pursue a demand side model in Online Appendix D below.

of the structure. This assumption is justified for the case of newly built houses and sales of vacant lots but it is less well justified for sales of properties with older structures where a demand side model may be more relevant.

There is a major practical problem with the hedonic regression model defined by (D.2): The multicollinearity problem. Experience has shown that it is usually not possible to estimate sensible land and structure prices in a hedonic regression like that defined by (D.2) due to the multicollinearity between lot size and structure size.[37] Thus in order to deal with the multicollinearity problem, the parameter β_t in (D.2) is replaced by p_{St}, an *exogenous period t construction cost price* for houses in the area under consideration.[38] The exogenous construction price index may be an official construction price index estimated by the national statistical agency or a relevant commercially available residential construction price index. Thus the new model that replaces (D.2) is the following nonlinear hedonic regression model:

$$V_{tn} = \alpha_t L_{tn} + p_{St}(1-\delta)^{A(t,n)} S_{tn} + \varepsilon_{tn}; \qquad t = 1, \ldots, T; n = 1, \ldots, N(t). \tag{D.3}$$

This model has T land price parameters (the α_t) and one (net) geometric depreciation rate δ. Note that the replacement of the β_t by the exogenous construction price level, p_{St}, means that we have saved T degrees of freedom as well as eliminated the multicollinearity problem.

In order to allow for a finer structure of local land prices, the sales data may be further classified into a finer classification of locations. For example, the initial regression (D.3) may be applied to say city wide sales of residential properties. Suppose that the postal code of each sale is also available and there are J postal codes. Then one can introduce the following *postal code dummy variables*, $D_{PC,tn,j}$, into the hedonic regression (D.3). These J dummy

[37] See Schwann (1998) and Diewert, de Haan and Hendriks (2011, 2015) on the multicollinearity problem.

[38] This formulation follows that of Diewert (2010), Diewert, de Haan and Hendriks (2011, 2015), Eurostat (2013), Diewert and Shimizu (2015, 2016, 2017a), Burnett-Isaacs, Huang and Diewert (2016) and Diewert, Huang and Burnett-Isaacs (2017). These authors assume that property value is the sum of land and structure components but movements in the price of structures are proportional to an exogenous structure price index. Note that the index p_{St} should be a levels price that gives the period t cost of building one square meter of structure.

variables are defined as follows: for $t = 1, \ldots, T; n = 1, \ldots, N(t); j = 1, \ldots, J$:

$$\begin{aligned} D_{PC,tn,j} &\equiv 1 \text{ if observation } n \text{ in period } t \text{ is in Postal Code } j; \\ &\equiv 0 \text{ if observation } n \text{ in period } t \text{ is } \textit{not} \text{ in Postal Code } j. \end{aligned} \tag{D.4}$$

We now modify the model defined by (D.3) to allow the *level* of land prices to differ across the J postal codes. The new nonlinear regression model is the following one:

$$V_{tn} = \alpha_t \left(\textstyle\sum_{j=1}^{J} \omega_j D_{PC,tn,j} \right) L_{tn} + p_{St}(1-\delta)^{A(t,n)} S_{tn} + \varepsilon_{tn}. \tag{D.5}$$

Here $t = 1, \ldots, T$ and $n = 1, \ldots, N(t)$.

Comparing the models defined by equations (D.3) and (D.5), it can be seen that we have added an additional J *neighbourhood relative land value parameters*, $\omega_1, \ldots, \omega_J$, to the model defined by (D.3). However, looking at (D.5), it can be seen that the T land time parameters (the α_t) and the J location parameters (the ω_j) cannot all be identified. Thus it is necessary to impose at least one identifying normalization on these parameters. The following normalization is a convenient one:[39]

$$\omega_1 \equiv 1. \tag{D.6}$$

Thus Model 2 is defined by equations (D.5) and (D.6) has $J - 1$ additional parameters compared to Model 1 defined by (D.3). Note that if we initially set *all* of the ω_j equal to unity, Model 2 collapses down to Model 1. It is useful to make use of this fact in running a sequence of nonlinear hedonic regressions. The models that are proposed in this section are *nested* so that the final parameter estimates from a previous model can be used as starting parameter values in the next model's nonlinear regression.[40]

[39] Equivalently, one could make the normalization $\alpha_1 = 1$ and not normalize the ω_j. The resulting estimated α_t for $t = 2, 3, \ldots, T$ can then be interpreted as a constant quality land price index for the entire region relative to period 1 where $\alpha_1 \equiv 1$. In this section, we are drawing heavily on Diewert, Huang and Burnett-Isaacs (2017) and using the normalization used in that paper.

[40] In order to obtain sensible parameter estimates in our final (quite complex) nonlinear regression model,

In the next model, some nonlinearities in the pricing of the land area for each property are introduced. The land plot areas in a typical sample of properties can vary 5 or 10 fold.[41] Up to this point, we have assumed that land plots in the same neighbourhood sell at a constant price per square meter of lot area. However, it is likely that there is some nonlinearity in this pricing schedule; for example, it is likely that large lots sell at a per m^2 price that is well below the per m^2 price of medium sized lots. In order to capture this nonlinearity, divide up the total number of observations into K groups of observations based on their lot size. The Group 1 properties have lot size less than L_1 m^2, the Group 2 properties L_{tn} have lot sizes which satisfy the inequalities $L_1 \leq L_{tn} < L_2$; the Group 3 properties L_{tn} have lot sizes which satisfy the inequalities $L_2 \leq L_{tn} < L_3$; ...; the Group K properties L_{tn} have lot sizes which satisfy the inequalities $L_{K-1} \leq L_{tn}$. The break points $L_1 < L_2 < \cdots < L_{K-1}$ should be chosen so that the sample probability that any property in the sample will fall into any one of the groups is approximately equal. For each observation n in period t, the K *land dummy variables*, $D_{L,tn,k}$, for $k = 1, \ldots, K$ are defined as follows:

$$\begin{aligned} D_{L,tn,k} &\equiv 1 \text{ if observation } tn \text{ has land area that belongs to group } k; \\ &\equiv 0 \text{ if observation } tn \text{ has land area that does not belong to group } k. \end{aligned} \tag{D.7}$$

These dummy variables are used in the definition of the following piecewise linear function

it is absolutely necessary to follow our procedure of sequentially estimating gradually more complex models, using the final coefficients from the previous model as starting values for the next model. The models that are being described in this section were implemented in Diewert, Huang and Burnett-Isaacs (2017) where the econometric software Shazam was used to perform the nonlinear regressions; see White (2004)[1].

[41] This brings up an important point that has not been mentioned up to now. Panel data on the selling prices of properties and on the characteristics of the properties are subject to tremendous variations in the ratio of the say highest price property to the lowest price property, to the largest lot size to the smallest lot size, to the largest floor space area to the smallest floor space area and so on. The observations that appear in the tales of the distribution of prices and in the distributions of property characteristics are inevitably sparse and subject to measurement error. Thus in order to obtain sensible estimates in running these hedonic regressions, it is typically necessary to delete the observations that are in the tales of these distributions.

of L_{tn}, $f_L(L_{tn})$, defined as follows:

$$\begin{aligned} f_L(L_{tn}) \equiv\ & D_{L,tn,1}\lambda_1 L_{tn} + D_{L,tn,2}[\lambda_1 L_1 + \lambda_2(L_{tn} - L_1)] \\ & + D_{L,tn,3}[\lambda_1 L_1 + \lambda_2(L_2 - L_1) + \lambda_3(L_{tn} - L_2)] \\ & + \cdots + D_{L,tn,K}[\lambda_1 L_1 + \lambda_2(L_2 - L_1) + \cdots + \lambda_K(L_{tn} - L_{K-1})], \end{aligned} \tag{D.8}$$

where the λ_k are unknown parameters. The function $f_L(L_{tn})$ defines a *relative valuation function for the land area of a house* as a function of the plot area, L_{tn}. The new nonlinear regression model is the following one:

$$V_{tn} = \alpha_t \left(\textstyle\sum_{j=1}^{J} \omega_j D_{PC,tn,j}\right) f_L(L_{tn}) + p_{St}(1-\delta)^{A(t,n)} S_{tn} + \varepsilon_{tn}. \tag{D.9}$$

Comparing the models defined by equations (D.5) and (D.9), it can be seen that we have added an additional K *land plot size parameters*, $\lambda_1, \ldots, \lambda_K$, to the model defined by (D.5). However, looking at (D.9), it can be seen that the T land time parameters (the α_t), the J postal code parameters (the ω_j) and the K land plot size parameters (the λ_k) cannot all be identified. Thus the following identification normalizations on the parameters for Model 3 defined by (D.9) and (D.10) are imposed:

$$\omega_1 \equiv 1; \lambda_1 \equiv 1. \tag{D.10}$$

Note that if all of the λ_k are set equal to unity, Model 3 collapses down to Model 2. Typically, the log likelihood for Model 3 will be considerably higher than for Model 2.[42] Land prices as functions of lot size do not always decline monotonically but for very large land plots, the marginal price of an extra square foot of land is typically quite low.

[42] For the example in Diewert, Huang and Burnett-Isaacs (2017) where the models described in this section were estimated, the log likelihood increased by 1762 log likelihood points and the R^2 jumped from 0.7662 for Model 2 to 0.8283 for Model 3 for the addition of 6 new λ_k parameters.

The next model is similar to Model 3 except that now the marginal price of adding an extra amount of structure is allowed to vary as the size of the structure increases. It is likely that the quality of the structure increases as the size of the structure increases. In order to capture this nonlinearity, divide up the sample observations into M groups of observations based on their structure size. The Group 1 properties have structures with floor space area S_{tn} less than S_1 m^2, the Group 2 properties have structure areas S_{tn} satisfying the inequalities $S_1 \leq S_{tn} < S_2$, ..., the Group $M-1$ properties have structure areas S_{tn} satisfying the inequalities $S_{M-2} \leq S_{tn} < S_{M-1}$, and the Group M properties have structure areas S_{tn} satisfying the inequalities $S_{M-1} \leq S_{tn}$ where the $M-1$ break points satisfy the inequalities $S_1 < S_2 < \cdots < S_{M-1}$. Again, the break points should be chosen so that the sample probability that any property in the sample will fall into any one of the groups is approximately equal. For each observation n in period t, we define the M *structure dummy variables*, $D_{S,tn,m}$, for $m = 1, \ldots, M$ as follows:

$$\begin{aligned} D_{S,tn,m} &\equiv 1 \text{ if obs. } tn \text{ has structure area in group } m; \\ &\equiv 0 \text{ otherwise.} \end{aligned} \tag{D.11}$$

These dummy variables are used in the definition of the following piecewise linear function of S_{tn}, $g_S(S_{tn})$, defined as follows:

$$\begin{aligned} g_S(S_{tn}) \equiv\; & D_{S,tn,1}\mu_1 S_{tn} + D_{S,tn,2}[\mu_1 S_1 + \mu_2(S_{tn} - S_1)] \\ & + D_{S,tn,3}[\mu_1 S_1 + \mu_2(S_2 - S_1) + \mu_3(S_{tn} - S_2)] \\ & + D_{S,tn,4}[\mu_1 S_1 + \mu_2(S_2 - S_1) + \mu_3(S_3 - S_2) + \mu_4(S_{tn} - S_3)] + \cdots \\ & + D_{S,tn,M}[\mu_1 S_1 + \mu_2(S_2 - S_1) + \mu_3(S_3 - S_2) + \cdots + \mu_M(S_{tn} - S_{M-1})]. \end{aligned} \tag{D.12}$$

where the μ_m are unknown parameters. The function $g_S(S_{tn})$ defines a *relative valuation function for the structure area of a house* as a function of the structure area.

The new nonlinear regression model is the following Model 4:

$$V_{tn} = \alpha_t \left(\sum_{j=1}^{J} \omega_j D_{PC,tn,j}\right) f_L(L_{tn}) + p_{St}(1-\delta)^{A(t,n)} g_S(S_{tn}) + \varepsilon_{tn};$$
$$t = 1,\ldots,T; n = 1,\ldots,N(t). \qquad \text{(D.13)}$$

Comparing the models defined by equations (D.9) and (D.13), it can be seen that an additional M *structure floor space parameters*, $\mu_1,\ldots,\mu_M$, have been added to the model defined by (D.9).[43] Again, we add the normalizations (D.10) in order to identify all of the parameters in the model. Note that if all of the μ_m are set equal to unity, Model 4 collapses down to Model 3. Typically, the log likelihood for Model 4 will be considerably higher than for Model 3.[44]

At this stage, it is often the case that an acceptable model has been estimated. How can the estimated parameters from the final model be used in order to form price and quantity indexes?

The sequence of price levels for the land component of residential property sales is defined to be $\alpha_1, \alpha_2, \ldots, \alpha_T$ and the corresponding sequence of price levels for the structure component of residential property sales in the T periods is defined to be the exogenous sequence of indexes, $p_{S1}, p_{S2}, \ldots, p_{ST}$. The land and structure values of properties transacted in period t, V_{Lt} and V_{St}, are defined by using the estimated land and structure additive components of transacted properties in period t, $\alpha_t(\sum_{j=1}^{J}\omega_j D_{PC,tn,j})f_L(L_{tn})$ and $p_{St}(1-\delta)^{A(t,n)}g_S(S_{tn})$

[43] At this stage of the sequential estimation procedure, it is usually not necessary to impose a normalization on the parameters μ_1-μ_M. This lack of a normalization means that the scale of the exogenous structure price levels p_{St} is allowed to change; i.e., essentially, allowance is now made to quality adjust the exogenous index to a certain extent. However, if the resulting estimated structure values turn out to be unreasonably large or small, then it will be necessary to set one of the μ_m to equal 1.

[44] For the example in Diewert, Huang and Burnett-Isaacs (2017), the log likelihood increased by 935 log likelihood points and the R^2 jumped from 0.8283 for Model 3 to 0.8520 for Model 4 for the addition of 5 new μ_M parameters.

respectively, and summing over properties that were sold in period t:

$$V_{Lt} \equiv \sum_{n \in N(t)} \alpha_t \left(\sum_{j=1}^{J} \omega_j D_{PC,tn,j} \right) f_L(L_{tn}); \qquad t = 1, \ldots, T; \tag{D.14}$$

$$V_{St} \equiv \sum_{n \in N(t)} p_{St} (1-\delta)^{A(t,n)} g_S(S_{tn}); \qquad t = 1, \ldots, T. \tag{D.15}$$

Using the prices $\alpha_1, \alpha_2, \ldots, \alpha_T$ and the corresponding estimated land values, $V_{L1}, \ldots, V_{LT}$ and the prices $p_{S1}, p_{S2}, \ldots, p_{ST}$ and the corresponding estimated structure values, $V_{S1}, \ldots, V_{ST}$, one can just apply normal index number theory using these data to construct Laspeyres, Paasche, Fisher or whatever index formula is being used by the statistical agency in order to construct constant quality price and quantity overall property indexes for the sales of residential properties in the area under consideration for the T periods.

However, constant quality land and structure price indexes for *sales* of owner-occupied Residential houses is not what is needed for most purposes; what is required are constant quality price and quantity indexes for the *stock* of residential houses. In order to accomplish this task, it is necessary to have a census of the housing stock in the country which would include information on the characteristics that are used in the hedonic regression model that is defined by (D.13). The information that is required in order to estimate (D.13) is information on the following variables:

- The selling price of the residential properties (P_{tn});
- The age of the structure on the property (A_{tn});
- The area of the land plot (L_{tn});
- The floor space area of the structure (S_{tn});
- The neighbourhood of the property (or the postal code) and
- An exogenous structure price index which provides the construction cost of a new structure per meter squared or per square foot (p_{St}).

If a national housing Census has information on the above property characteristics (excluding the information on selling prices P_{tn} and on the exogenous structure price index p_{St})[45] then it will be possible to insert the characteristics of each residential dwelling unit into the right hand side of (D.13) and then using appropriate modifications of definitions (D.14) and (D.15), it will be possible to obtain estimates for the land and structure value for each dwelling unit in the area covered by the regression. If there is no national housing census information or the required characteristics are not included in the census, then it will be very difficult to form estimates for the value of residential land.

Additional information on house and property characteristics will lead to more accurate land and structure decompositions of property value. Examples of useful additional structure price determining characteristics are: (i) the number of bathrooms; (ii) the number of bedrooms; (iii) the type of construction material; (iv) the number of stories; etc. Examples of useful additional land price determining characteristics are: (i) the distance to the nearest subway station; (ii) the distance to the city core; (iii) the quality of neighbourhood schools; (iv) the existence of various neighbourhood amenities; etc. For examples of how these characteristics can be integrated into the builder's model, see Diewert, de Haan and Hendriks (2011) 2015, Eurostat (2013, 2017), Diewert and Shimizu (2015) and Diewert, Huang and Burnett-Isaacs (2017).[46]

The estimates for the geometric depreciation rate generated by the application of the builder's model are useful for national income accountants because they facilitate the accurate estimation of structure depreciation, which is required for the national accounts. However, the depreciation estimates that are generated by the builder's model are *wear and tear depreciation* estimates that apply to structures that continue in existence over the sample period.

[45] Every country will have a national residential construction deflator because this deflator is required in order to form estimates of real investment in residential structures. However, this national deflator may not be entirely appropriate for the type of buildings in a particular neighbourhood.

[46] It is also possible to estimate more general models of depreciation using the builder's model; see Diewert and Shimizu (2017a) and Diewert, Huang and Burnett-Isaacs (2017).

The estimated depreciation rate measures (net) depreciation[47] of a structure that has survived from its birth to the period of its sale. However, there is another form of structure depreciation that the estimated depreciation rate misses; namely the loss of residual structure value that results from the *early demolition* of the structure. This problem was noticed and addressed by Hulten and Wykoff (1981a, pp. 377–379) 1981b, 1996. Wear and tear depreciation is often called *deterioration* depreciation and *demolition* or *early retirement depreciation* is sometimes called *obsolescence* depreciation.[48] Methods for estimating this form of depreciation have been proposed by Hulten and Wykoff as mentioned above and by Diewert and Shimizu (2017a, pp. 512–516). Both methods require information on the distribution of the ages of retirement for the asset class. The Hulten and Wykoff method absorbs demolition depreciation into the wear and tear depreciation rate whereas the Diewert and Shimizu method uses the wear and tear depreciation rate that is generated by sales of surviving buildings but adds a separate depreciation rate that is due to early demolition of the structures in the asset class. Both methods require information on the age of structures when they are demolished.[49]

The above paragraph simply warns the reader that wear and tear depreciation[50] for surviving buildings is not the entire depreciation story: there is also a loss of asset value that results

[47]It is a net estimate since renovation and replacement investments in the building tend to extend the life of the building or augment its value. Thus the gross wear and tear depreciation rate for the structure will tend to be larger than the estimated net depreciation rate.

[48]Crosby, Devaney and Law (2012); 230 distinguish the two types of depreciation and in addition, they provide a comprehensive survey of the depreciation literature as it applies to commercial properties.

[49]The Hulten and Wykoff method estimates the age of retirement in a somewhat arbitrary fashion whereas the Diewert and Shimizu method relies on mortality distributions on the age of buildings at the time they are demolished. Over long periods of time and using country wide data, the two methods should be equivalent. However, the Diewert and Shimizu method should give more accurate results at the firm and regional levels since their method is consistent with the hedonic estimation of structure depreciation rates as explained in this section.

[50]What has been labeled as wear and tear depreciation could be better described as *anticipated amortization of the structure* rather than wear and tear depreciation. Once a structure is built, it becomes a fixed asset which cannot be transferred to alternative uses (like a truck or machine). Thus amortization of the cost of the structure should be proportional to the cash flows or to the service flows of utility that the building generates over its expected lifetime. However, technical progress, obsolescence or unanticipated market developments can cause the building to be demolished before it is fully amortized. See Diewert and Fox (2016) for a more complete discussion of the fixity problem.

from the early retirement of a building that needs to be taken into account when constructing national income accounting estimates of depreciation.

There is one additional complication that needs to be taken into account when running a hedonic regression on the sales of houses; i.e., what happens when the sales information for an additional period becomes available? The simplest way of dealing with this problem dates back to Court (1939). His method works as follows: set $T = 2$ and run a hedonic regression that has a time dummy variable in it. In the context of the hedonic regression model defined by (D.13), estimates for the price of land for periods 1 and 2 would be obtained, say α_1^1 and α_2^1. The price index for land for periods 1 and 2 is defined as $P_L^1 = 1$ and $P_L^2 = \alpha_2^1/\alpha_1^1$. Now run a new hedonic regression using (D.13) for $t = 2, 3$ and obtain new estimates for the price of land in periods 2 and 3, say α_2^2 and α_3^2. The price index for land in period 3 is defined as $P_L^3 = P_L^2(\alpha_3^2/\alpha_2^2)$; i.e., we update the price index value for period 2, P_L^2, by the rate of change in land prices going from period 2 to 3, (α_3^2/α_2^2). Thus the previously estimated index is updated each period as new information becomes available. This *adjacent period time dummy model* has the advantage that it does not revise the previously estimated indexes as the new information becomes available.[51]

The above method does not always work well in the context of estimating property price indexes due to the sparseness of sales in a neighbourhood and the multiplicity of parameters that are required to adequately control for differences in housing characteristics. Thus Shimizu, Nishimura and Watanabe 2010a, p. 797 suggested extending the number of periods from 2 to a longer window of T consecutive periods, leading to the *rolling window time dummy hedonic regression model*. Thus for the model defined by (D.13), the land price

[51] The two period time dummy variable hedonic regression (and its extension to many periods) was first considered explicitly by Court (1939); 109-111 as his hedonic suggestion number two. Court used adjacent period time dummy hedonic regressions as links in a longer chain of comparisons extending from 1920 to 1939 for US automobiles: "The net regressions on time shown above are in effect price link relatives for cars of constant specifications. By joining these together, a continuous index is secured." If the two periods being compared are consecutive years, Griliches (1971, p. 7) coined the term "adjacent year regression" to describe this method for updating the index as new information becomes available. Diewert (2005b) looked at the axiomatic properties of adjacent year time dummy hedonic regressions.

parameters that are estimated by the first regression using the data for periods 1 to T are $\alpha_1^1, \alpha_2^1, \ldots, \alpha_T^1$ and the corresponding land price indexes for periods 1 to t are $P_L^t \equiv \alpha_t^1/\alpha_1^1$ for $t = 1, \ldots, T$. The second hedonic regression uses the data for periods $2, 3, \ldots, T, T+1$ and the estimated land price parameters are $\alpha_2^2, \alpha_3^2, \ldots, \alpha_T^2, \alpha_{T+1}^2$. The price index for land in period $T+1$ is defined as $P_L^{T+1} = P_L^T(\alpha_{T+1}^2/\alpha_T^2)$; i.e., the price index for period T, P_L^t, is updated by the rate of change in land prices going from period T to $T+1$, $\alpha_{T+1}^2/\alpha_T^2$.

There are two additional issues that need to be addressed when using a rolling window time dummy hedonic regression model:

- How long should the window length be? A longer window length will probably lead to more stable estimates for the unknown parameters in the hedonic regression. A shorter window length will allow for taste changes to take place more quickly. A window length of one year plus one period will allow for seasonal effects. At this stage of our knowledge, it is difficult to give definitive advice on the length of the window.

- When a new window is computed, how should the index results from the new window be linked to the previous index values? The same issue applies when a multilateral method is used in the time series context. Ivancic, Diewert and Fox (2011) along with Shimizu, Nishimura and Watanabe (2010a) and Shimizu, Takatsuji, Ono and Nishimura (2010) suggested that the movement of the indexes for the last two periods in the new window be linked to the last index value generated by the previous window. However Krsinich (2016) suggested that the movement of the indexes generated by the new window over the entire new window period be linked to the previous window index value for the second period in the previous window. Krsinich called this a *window splice* as opposed to the *movement splice* explained above. de Haan (2015); 27 suggested that perhaps the linking period should be in the middle of the old window which the Australian Bureau of Statistics (2016); 12 termed a *half splice*. Ivancic, Diewert and Fox (2011); 33 suggested that the average of all possible links of the new window to the old window be used and they called this a *mean splice* method for linking the results of the new window to the previous window.[52] Again, there is no consensus at this time on which linking method is "best". However, it is likely that all of these linking methods will generate much the same results.

It can be seen that estimating price indexes for houses (or detached dwelling units) is not a straightforward task, particularly if one wants separate constant quality indexes for the land and structure components of property value.[53] In the following section, it will be seen that it is even more complicated to obtain separate indexes for the land and structure components for condominium sales.

D.1 Decomposing Condominium Sales Prices into Land and Structure Components

A starting point for applying the builder's model to condominium sales is the hedonic regression model defined by equations (D.3) in the previous section.[54] For convenience, equations (D.3) are repeated below as equations (D.16):

$$V_{tn} = \alpha_t L_{tn} + p_{St}(1-\delta)^{A(t,n)} S_{tn} + \varepsilon_{tn}; \qquad t = 1, \ldots, T; n = 1, \ldots, N(t), \tag{D.16}$$

where V_{tn} is the selling price of a condominium property in a neighbourhood in period t, α_t is the price of the land that the structure sits on (per m^2), L_{tn} is the land area that can be attributed to the condo unit, p_{St} is an exogenous period t construction cost for the type of condo under consideration (per m^2), δ is the one period wear and tear geometric depreciation rate for the structure, $A_{tn} = A(t,n)$ is the age of the structure in periods, S_{tn} is the floor

[52] For the details on how the mean splice method works, see Diewert and Fox (2017).

[53] For additional hedonic regression models for detached houses, see Verbrugge (2008), Garner and Verbrugge (2011), Eurostat (2013, 2017), Hill (2013), Hill, Scholz, Shimizu and Steurer (2018), Rambaldi and Fletcher (2014) and Silver (2018).

[54] The analysis in this section follows that of Diewert and Shimizu (2016).

space of unit n that is sold in period t (in m^2) and ε_{tn} is an error term.

A problem with the above model is that it is not appropriate to allocate the entire land value of the condominium property to any particular unit that is sold in period t. Thus each condo unit in the building should be allocated a *share* of the total land value of the property. The problem is: how exactly should this imputed land share be calculated? There are two simple methods for constructing an appropriate land share: (i) Use the unit's share of floor space to total structure floor space or (ii) simply use $1/N$ as the share where N is the total number of units in the building. Thus define the following two land share imputations for unit n in period t:

$$L_{Stn} \equiv (S_{tn}/TS_{tn})TL_{tn},$$
$$L_{Ntn} \equiv (1/N_{tn})TL_{tn}; \quad t = 1, \ldots, T;\ n = 1, \ldots, N(t). \tag{D.17}$$

where S_{tn} is the floor space area of unit n which is sold in period t, TS_{tn} is the total building floor space area, TL_{tn} is the total land area of the building and N_{tn} is the total number of units in the building for unit n sold in period t. The first method of land share imputation is used by the Japanese land tax authorities. The second method of imputation implicitly assumes that each unit can enjoy the use of the entire land area and so an equal share of land for each unit seems "fair".

There is a problem with the definition of L_{Stn} in (D.17): the floor space "share" of unit n, S_{tn}/TS_{tn} if summed over all units in the building would be less than 1 because the privately held floor space of each unit in the building does not account for shared building floor spaces such as halls, elevators, storage spaces, furnace rooms and other "public" floor spaces, which are included in total building floor space, TS_{tn}. Thus the "share" S_{tn}/TS_{tn} must be adjusted upward by some percentage to account for these shared building facilities.[55] In what follows,

[55] Diewert and Shimizu (2016, p. 303) constructed estimates of Tokyo total building private floor space to total building floor space for each observation nt as $N_{tn}S_{tn}/TS_{tn}$, where N_{tn} is the number of units in the building which contained condo sale n in period t, S_{tn} is the private floor space of the sold unit and

it is assumed that this adjustment has been made to S_{tn} (so that S_{tn} is now interpreted as adjusted condo floor space area).

In order to obtain sensible decompositions of the condominium selling price into land and structure components, it may be necessary to assume a structure value and focus on the determinants of land value at the initial stages of the sequential estimation procedure. Thus following Diewert and Shimizu (2016), assume that the *imputed structure value* for unit n in period t, V_{Stn}, is defined as follows:

$$V_{Stn} \equiv p_{St}(1-\delta)^{A(t,n)}S_{tn}; \qquad t = 1,\ldots,T; n = 1,\ldots,N(t), \tag{D.18}$$

where δ is an assumed geometric depreciation rate.[56] Once the imputed value of the structure has been defined by (D.18), the *imputed land value* for condo n in period t, V_{Ltn}, is defined by subtracting the imputed structure value from the total value of the condo unit, which is V_{tn}:

$$V_{Ltn} \equiv V_{tn} - V_{Stn}; \qquad t = 1,\ldots,T; n = 1,\ldots,N(t). \tag{D.19}$$

In the hedonic regressions which follow immediately, the imputed value of land for the condominium unit, V_{Ltn}, is used as the dependent variable in a hedonic regression. The following regressions explain variations in these imputed land values in terms of the property characteristics.

Suppose that the postal code of each sale is also available and there are J postal codes. Then one can introduce the following *postal code dummy variables*, $D_{PC,tn,j}$, as explanatory variables into a hedonic regression. Define these J dummy variables using definitions (D.4) in

TS_{tn} is the total floor space of the building. The sample wide average of these ratios was 0.899. Thus the first imputation method in definitions (D.17) was changed from $L_{Stn} \equiv (S_{tn}/TS_{tn})TL_{tn}$ to $L_{Stn} \equiv (1/0.899)(S_{tn}/TS_{tn})TL_{tn} = (1.1)(S_{tn}/TS_{tn})TL_{tn}$. Burnett-Isaacs, Huang and Diewert (2016) estimated a similar condo model and consulted with construction experts and determined that on average, the ratio of total space to private space for Ottawa condominium apartments was approximately 1.33. Thus they changed $L_{Stn} \equiv (S_{tn}/TS_{tn})TL_{tn}$ to $L_{Stn} \equiv (1.33)(S_{tn}/TS_{tn})TL_{tn}$.

[56] Diewert and Shimizu (2016) assumed $\delta = 0.03$ and Burnett-Isaacs, Huang and Diewert (2016) assumed $\delta = 0.02$ where the age variable A_{tn} is measured in years. Later, δ will be estimated.

the previous section and estimate the following hedonic regression which is a *land counterpart* to the hedonic regression defined by (D.5) in the previous section: are defined as follows:

$$V_{Ltn} = \alpha_t \left(\sum_{j=1}^{J} \omega_j D_{PC,tn,j}\right) L_{Stn} + \varepsilon_{tn}; \qquad t = 1, \ldots, T; n = 1, \ldots, N(t). \tag{D.20}$$

Note that the imputed value of land, V_{Ltn} defined by (D.19), replaces total property value V_{tn} which was the dependent variable in (D.5).[57]

It is likely that the height of the building (number of stories) increases the value of the land plot supporting the building, all else equal. Thus define the number of stories dummy variables, $D_{NS,tn,s}$, as follows: $t = 1, \ldots, T; n = 1, \ldots, N(t); s = 1, \ldots, NS$:

$$\begin{aligned} D_{NS,tn,s} &\equiv 1 \text{ if observation } n \text{ in period } t \text{ is in a building with } s \text{ stories ;} \\ &\equiv 0 \text{ if observation } n \text{ in period } t \text{ is } \textit{not} \text{ in building with } s \text{ stories.} \end{aligned} \tag{D.21}$$

The new nonlinear regression model is the following one:

$$V_{Ltn} = \alpha_t \left(\sum_{j=1}^{J} \omega_j D_{PC,tn,j}\right) \left(\sum_{s=1}^{NS} \chi_s D_{NS,tn,s}\right) L_{Stn} + \varepsilon_{tn}, \tag{D.22}$$

Comparing the models defined by equations (D.20) and (D.22), it can be seen that an additional NS *building height parameters*, $\chi_1, \ldots, \chi_{NS}$, have been added to the model defined by (D.20).[58] As usual, the models defined by (D.20) and (D.22) are nested so that the finishing parameter values from the nonlinear regression (D.20) can be used as starting values for (D.22) along with the starting values $\chi_1 = \chi_2 = \cdots = \chi_{NS} = 1$.

The higher up a unit is, the better is the view on average and so it could be expected that

[57] As usual, we need a normalization on the parameters such as $\alpha_1 = 1$ in order to identify all of the remaining parameters, $\alpha_2, \ldots, \alpha_T, \omega_1, \ldots, \omega_J$. Note that this regression uses the first method of land imputation defined by (D.17). Later, the second method will also be considered.

[58] Again normalizations like $\alpha_1 \equiv 1; \chi_1 \equiv 1$ are required in order to identify the remaining parameters. If all $\chi_s = 1$, then the model defined by (D.22) collapses down to the model defined by (D.20).

the price of the unit increases as its height increases. The quality of the structure probably does not increase as the height of the unit increases so it seems reasonable to impute the height premium as an adjustment to the land price component of the unit.

It is possible to introduce the height of the unit (the H variable) as a categorical variable (like the number of stories NS in the last hedonic regression model). However, both Diewert and Shimizu (2016) (hereafter DS) and Burnett-Isaacs, Huang and Diewert (2016) (hereafter BHD) found that this dummy variable approach could be replaced by using H as a continuous variable with little change in the fit of the model. Thus the new nonlinear regression model is the following one where $t = 1, \ldots, T; n = 1, \ldots, N(t)$:

$$V_{Ltn} = \alpha_t \left(\sum_{j=1}^{J} \omega_j D_{PC,tn,j}\right) \left(\sum_{s=1}^{NS} \chi_s D_{NS,tn,s}\right) (1 + \gamma(H_{tn} - 3)) L_{Stn} + \varepsilon_{tn}; \qquad \text{(D.23)}$$

where H_{tn} is the height of the sold unit n in period t (measured in number of stories from ground level) and γ is a height of the unit parameter to be estimated.[59] The above model assumes that the lowest height for the units sold in the sample was $H_{tn} = 3$. Thus for all the observations that correspond to the sold unit being located on the third floor of the building, the new parameter γ in (D.23) will not affect the predicted value in the regression. However, for heights of the sold units that were greater than 3, the regression implies that the land value will increase by γ for each story that is above 3.[60]

As was mentioned earlier, there are two simple methods for imputing the share of the building's total land area to the sold unit. Up until now, we have used the first method of imputation defined by (D.17) which set the share of total land imputed to unit n in period t, L_{Stn}, equal to $(S_{tn}/TS_{tn})TL_{tn}$ whereas the second method set L_{Ntn} equal to $(1/N_{tn})TL_{tn}$. In the next model, the land imputation for unit n in period t is set equal to a *weighted average*

[59] Normalizations like $\alpha_1 \equiv 1; \chi_1 \equiv 1$ need to be imposed in order to identify the remaining parameters.

[60] The studies that have implemented this model found that the estimated γ was in the 2-4% range. Thus the imputed land value of a unit increases by 2 to 4% for each story above the threshold level of 3.

of the two imputation methods and the best fitting weight, λ, is estimated. Thus define:

$$L_{tn}(\lambda) = [\lambda(S_{tn}/TS_{tn}) + (1-\lambda)(1/N_{tn})]TL_{tn}; \qquad t = 1, \ldots, T; n = 1, \ldots, N(t). \quad \text{(D.24)}$$

The new nonlinear regression model is the following one where $t = 1, \ldots, T; n = 1, \ldots, N(t)$ and $L_{tn}(\lambda)$ is defined by (D.24).[61]:

$$V_{Ltn} = \alpha_t \left(\sum_{j=1}^{J} \omega_j D_{PC,tn,j}\right) \left(\sum_{s=1}^{NS} \chi_s D_{NS,tn,s}\right) (1 + \gamma(H_{tn} - 3))L_{tn}(\lambda) + \varepsilon_{tn}. \quad \text{(D.25)}$$

Conditional on the land area of the building, one would expect the sold unit's land imputation value to increase as the number of units in the building increases. Thus one could use the total number of units in the building, N_{tn}, as a quality adjustment variable for the imputed land value of a condo unit. DS introduced this variable as a continuous variable. The smallest number of units in the buildings in their sample was 11. Thus they introduced the term $1+\kappa(N_{tn}-11)$ as an explanatory term in the nonlinear regression. The new parameter κ is the percentage increase in the unit's imputed value of land as the number of units in the building grows by one unit. The new nonlinear regression model is the following one where $t = 1, \ldots, T; n = 1, \ldots, N(t)$ and $L_{tn}(\lambda)$ is defined by (D.24):

$$V_{Ltn} = \alpha_t \left(\sum_{j=1}^{J} \omega_j D_{PC,tn,j}\right) \left(\sum_{s=1}^{NS} \chi_s D_{NS,tn,s}\right) \cdot (1 + \gamma(H_{tn} - 3))(1 + \kappa(N_{tn} - 11))L_{tn}(\lambda) + \varepsilon_{tn}. \quad \text{(D.26)}$$

where $L_{tn}(\lambda)$ is defined by (D.24).

The next explanatory variable to be introduced into the hedonic regression model is one which is not obvious but turned out to be very significant in the regressions run by DS and

[61] For the DS Tokyo condo data, the estimated λ turned out to be $\lambda^* = 0.3636$ ($t = 9.84$) so that the very simple land imputation method that just divided the total land plot size by the number of units in the building got a higher weight (0.6364) than the weight for the floor space allocation method (0.3636). For the Ottawa condo data, the estimated λ turned out to be $\lambda^* = 0.2525$ ($t = 12.10$).

BHD. The *footprint* of a building is the area of the land that directly supports the structure. An approximation to the footprint land for unit n in period t is the total structure area TS_{tn} divided by the total number of stories in the structure TH_{tn}. If footprint land is subtracted from the total land area, TL_{tn}, the resulting variable is *excess land*,[62] EL_{tn}, defined as follows:

$$EL_{tn} \equiv TL_{tn} - (TS_{tn}/TH_{tn}); \qquad t = 1, \ldots, T; n = 1, \ldots, N(t). \tag{D.27}$$

In the Tokyo data used by DS, excess land ranged from 47 m^2 to 2912 m^2. Now group the sample observations into M categories, depending on the amount of excess land that pertained to each observation. Group 1 consists of observations tn where EL_{tn} is less than some number EL_1; Group 2: observations such that $EL_1 \leq EL_{tn} < EL_2$; … ; Group M: $EL_{M-1} \leq EL_{tn}$. The break points, $EL_1, EL_2, \ldots, EL_{M-1}$ should be chosen so that the number of observations in each group is approximately equal. Define the *excess land dummy variables*, $D_{EL,tn,m}$, as follows for $t = 1, \ldots, T; n = 1, \ldots, N(t); m = 1, \ldots, M$:

$$\begin{aligned} D_{EL,tn,m} &\equiv 1 \text{ if observation } n \text{ in period } t \text{ is in excess land group } m; \\ &\equiv 0 \text{ if observation } n \text{ in period } t \text{ is } \textit{not} \text{ in excess land group } m. \end{aligned} \tag{D.28}$$

The new regression model is the following one:

$$\begin{aligned} V_{Ltn} = \alpha_t \left(\textstyle\sum_{j=1}^{J} \omega_j D_{PC,tn,j}\right) \left(\textstyle\sum_{s=1}^{NS} \chi_s D_{NS,tn,s}\right) \left(\textstyle\sum_{m=1}^{M} \mu_m D_{EL,tn,m}\right) \times \\ (1 + \gamma(H_{tn} - 3))(1 + \kappa(N_{tn} - 11)) L_{tn}(\lambda) + \varepsilon_{tn}; \\ t = 1, \ldots, T; \; n = 1, \ldots, N(t). \end{aligned} \tag{D.29}$$

Not all of the parameters in (D.29) can be identified so the following normalizations on the

[62] This is land that is usable for purposes *other* than the direct support of the structure on the land plot.

parameters in (D.29) are imposed:

$$\alpha_1 \equiv 1; \chi_1 \equiv 1; \mu_1 \equiv 1. \tag{D.30}$$

Introducing the excess land dummy variables led to huge jumps in the log likelihoods for the hedonic regressions run by DS and BHS: 1020 for DS and 2652 for BHS.[63] Both studies found that the estimated μ_m were positive but their magnitudes decreased monotonically as the excess land variable increased.

There are three additional explanatory variables that were used by DS that may affect the price of land. Define TW as the walking time in minutes to the nearest subway station; TT as the subway running time in minutes to the Central Tokyo station from the nearest station and the SOUTH dummy variable is set equal to 1 if the sold condo unit faces south and 0 otherwise. Let $D_{S,tn,2}$ equal the SOUTH dummy variable for sale n in period t. Define $D_{S,tn,2} = 1 - D_{S,tn,1}$. In the Tokyo data set used by DS, TW ranged from 1 to 19 minutes while TT ranged from 12 to 48 minutes. These new variables are inserted into the previous nonlinear regression model (D.29) in the following manner for $t = 1, \ldots, T; n = 1, \ldots, N(t)$:

$$\begin{aligned} V_{Ltn} = \alpha_t &\left(\textstyle\sum_{j=1}^{J} \omega_j D_{PC,tn,j}\right) \left(\textstyle\sum_{s=1}^{NS} \chi_s D_{NS,tn,s}\right) \left(\textstyle\sum_{m=1}^{M} \mu_m D_{EL,tn,m}\right) \times \\ &(\phi_1 D_{S,tn,1} + \phi_2 D_{S,tn,2})(1 + \gamma(H_{tn} - 3))(1 + \kappa(N_{tn} - 11)) \times \\ &(1 + \eta(TW_{tn} - 1))(1 + \theta(TT_{tn} - 12))L_{tn}(\lambda) + \varepsilon_{tn}; \end{aligned} \tag{D.31}$$

where $L_{tn}(\lambda)$ is defined by (D.24). Not all of the parameters in (D.31) can be identified so the following normalizations (D.32) are imposed on the parameters in (D.31):

$$\alpha_1 \equiv 1; \chi_1 \equiv 1; \mu_1 \equiv 1; \phi_1 \equiv 1. \tag{D.32}$$

[63]Recall the hedonic regression model defined by (D.9) in the previous section which introduced linear splines on the valuation of the land area of a stand alone housing unit. This introduction also greatly increased the log likelihood of the regression. In the present context, the excess land dummy variables take the place of the linear spline functions in (D.9).

Using the DS Tokyo data, the R^2 for this model turned out to be 0.6308 and the log likelihood increased by 406 points over the log likelihood of the previous model defined by (D.29) for the addition of 3 new parameters. The estimated parameters had the expected signs and had reasonable magnitudes.

At this point, DS concluded that the imputed land value for each condominium in their sample was predicted reasonably well by the hedonic regression defined by (D.31) and (D.32). Thus in the following regression, they switched from using the imputed land value V_{Ltn} defined by (D.19) as the dependent variable in the regressions to using the actual selling price of the property, V_{tn}. They used the specification for the land component of the property that that is defined by (D.31) and (D.32) but they also added the structure term $p_{St}(1-\delta)^{A(t,n)}S_{tn}$ to account for the structure component of the value of the condo unit. Note that the annual depreciation rate δ is now estimated by the new hedonic regression model, rather than assuming that it was equal to 3%. Thus the number of unknown parameters in the new model increased by 1. They used the estimated values for the coefficients in (D.31) as starting values in this new nonlinear regression.[64]

Using their Tokyo data, DS found that the R^2 for this new model was 0.8190 and the estimated depreciation rate was $\delta^* = 0.0367$ ($t = 27.1$). Note that the R^2 is satisfactory; i.e., the new model explains a substantial fraction of the variation in condo prices.

DS and BHD introduced some additional explanatory variables as quality adjusting variables for the imputed value of structures. DS introduced the number of bedrooms and the type of building as quality adjusters for the value of the structure. BHD introduced the number of bedrooms, the number of bathrooms, the presence of balconies, the use of natural gas as the heating fuel and whether there was commercial space in the building as additional

[64] Attempting to estimate the parameters in (D.33) without good starting values for the nonlinear regression will not lead to sensible parameter estimates. Thus it is necessary to obtain good starting values for (D.33) by estimating the rather long sequence of regressions explained above, starting with a very simple model and gradually introducing additional explanatory variables. Each regression in the sequence contains the previous one as a special case so that the final estimates of one regression can be used as starting values for the subsequent one.

variables that could determine the value of the structure. These variables were significant explanatory variables but the overall R^2 for the final hedonic regression did not increase by a large amount with the addition of these variables to the regression. The details may be found in Diewert and Shimizu (2016) and Burnett-Isaacs, Huang and Diewert (2016).

Once the final hedonic regression has been run, the sequence of land prices is given by $\alpha_1, \alpha_2, \ldots, \alpha_T$ and the sequence of condo structure prices is given by the exogenous structure price indexes, $p_{S1}, p_{S2}, \ldots, p_{ST}$. To obtain overall property price indexes for sales of condos, form the following counterparts to equations (D.14) and (D.15) in the previous section to obtain an estimate of period t condo land value, V_{Lt}, and estimated period t structure value, V_{St}, for $t = 1, \ldots, T$:

$$V_{Lt} \equiv \textstyle\sum_{n\in N(t)} \alpha_t \left(\sum_{j=1}^{J} \omega_j D_{PC,tn,j}\right) \left(\sum_{s=1}^{NS} \chi_s D_{NS,tn,s}\right) \left(\sum_{m=1}^{M} \mu_m D_{EL,tn,m}\right) \times$$
$$(\phi_1 D_{S,tn,1} + \phi_2 D_{S,tn,2})(1 + \gamma(H_{tn} - 3))(1 + \kappa(N_{tn} - 11))\times$$
$$(1 + \eta(TW_{tn} - 1))(1 + \theta(TT_{tn} - 12))L_{tn}(\lambda); \tag{D.33}$$

$$V_{St} \equiv \textstyle\sum_{n\in N(t)} \quad p_{St}(1 - \delta)^{A(t,n)} S_{tn}. \tag{D.34}$$

Using the prices $\alpha_1, \alpha_2, \ldots, \alpha_T$ and the corresponding estimated land values, $V_{L1}, \ldots, V_{LT}$ and the prices $p_{S1}, p_{S2}, \ldots, p_{ST}$ and the corresponding estimated structure values, $V_{S1}, \ldots, V_{ST}$, one can again apply normal index number theory using these data to construct Laspeyres, Paasche, Fisher or whatever index formula is being used by the statistical agency in order to construct constant quality price and quantity overall property indexes for the sales of condominium units in the area under consideration for the T periods.

In summary: the builder's model can be modified to apply to the sales of condominium units and reasonable decompositions of property value into land and structure components can be obtained. However, the nonlinear regressions that are required in order to implement the model end up being rather complex. In addition, information on more characteristics of the condominium properties needs to be collected in order to implement the models. The information that is required in order to estimate the final model and calculate (D.33) and (D.34) is as follows:

- The selling prices of the condominium properties in the sample (P_{tn});
- The age of the structure on the property (A_{tn});
- The total area of the land plot (TL_{tn});
- The floor space area of the condo unit (S_{tn});
- The total floor space area of the entire building (TS_{tn});
- The neighbourhood of the property (or the postal code);
- An exogenous structure price index which provides the construction cost of a new structure per meter squared or per square foot (p_{St});
- The number of stories of the building (NS_{tn});
- The height of the sold unit (the number of stories from ground level) (H_{tn});
- The number of units in the building (N_{tn});
- The walking time in minutes to the nearest subway station (TW_{tn}) and
- The subway running time in minutes to the city center from the nearest station (TT_{tn}).

The last two variables are not essential (and are not relevant in small towns and cities). Other non-essential variables which could be useful are the number of bedrooms, the number of bathrooms, the existence of balconies, the type of construction, the number of parking spaces and so on.

The hedonic regression models that were considered in the last two sections are essentially modified supply side models. In the following section, demand side hedonic regressions are considered.

D.2 Demand Side Property Price Hedonic Regressions

A way of rationalizing the traditional log price time dummy hedonic regression model for properties with varying amounts of land area L and constant quality structure area S^* is that the utility that these properties yield to consumers is proportional to the Cobb-Douglas utility function $L^\alpha S^{*\beta}$ where α and β are positive parameters (which do not necessarily sum to one).[65] Initially, assume that the constant quality structure area S^* is equal to the floor space area of the structure, S, times an age adjustment, $(1-\delta)^A$, where A is the age of the structure in years and δ is a positive depreciation rate that is less than 1. Thus S^* is related to S as follows:

$$S^* \equiv S(1-\delta)^A. \tag{D.35}$$

In any given time period t, assume that the sale price of transacted property n, V_{tn}, with the amount of land L_{tn} and the amount of quality adjusted structure S^*_{tn} is equal to the following expression:

$$\begin{aligned} V_{tn} &= p_t L_{tn}^\alpha S_{tn}^{*\beta} \\ &= p_t L_{tn}^\alpha [S_{tn}(1-\delta)^{A(t,n)}]^\beta \qquad \text{using (D.35)} \\ &= p_t L_{tn}^\alpha S_{tn}^\beta (1-\delta)^{\beta A(t,n)} \\ &= p_t L_{tn}^\alpha S_{tn}^\beta \phi^{A(t,n)}, \end{aligned} \tag{D.36}$$

where $A(t,n) = A_{tn}$ is the age of house n sold in period t, p_t can be interpreted as a *period t property price index* and the constant ϕ is defined as follows:

$$\phi \equiv (1-\delta)^\beta. \tag{D.37}$$

[65] The early analysis in this section follows that of McMillen (2003, pp. 289–290), Shimizu, Nishimura and Watanabe (2010a, p. 795) and Diewert, Huang and Burnett-Isaacs (2017). McMillen assumed that $\alpha+\beta = 1$. We follow Shimizu, Nishimura and Watanabe in allowing α and β to be unrestricted.

Thus if V_{tn} is deflated by the period t property price index p_t, the real value or utility u_{tn} of the property with characteristics L_{tn} and S^*_{tn} is obtained:

$$V_{tn}/p_t = L^{\alpha}_{tn} S^{*\beta}_{tn} \equiv u_{tn}. \tag{D.38}$$

Thus $u_{tn} \equiv q_t$ is the *aggregate real value of the property* with characteristics L_{tn} and S^*_{tn}.

Define ρ_t as the logarithm of p_t and γ as the logarithm of ϕ; i.e.,

$$\rho_t \equiv \ln p_t; \gamma \equiv \ln \phi. \tag{D.39}$$

After taking logarithms of both sides of the first equation in (D.38), using definitions (D.35) and (D.39) and adding error terms, the following system of estimating equations is obtained:[66]

$$\ln V_{tn} = \rho_t + \alpha \ln L_{tn} + \beta \ln S_{tn} + \gamma A_{tn} + \varepsilon_{tn}; \qquad t = 1, \ldots, T; n = 1, \ldots, N(t), \tag{D.40}$$

where the ε_{tn} are independently distributed error terms with 0 means and constant variances. It can be seen that (D.40) is a traditional log price time dummy hedonic regression model with a minimal number of characteristics. The unknown parameters in (D.40) are the constant quality log property prices, $\rho_1, \ldots, \rho_T$, and the taste parameters α, β and the transformed depreciation rate γ. Once these parameters have been determined, the geometric depreciation rate δ which appears in equations (D.36) can be recovered from the regression parameter estimates as follows:

$$\delta \equiv 1 - e^{\gamma/\beta}. \tag{D.41}$$

We now explain how the hedonic pricing model defined by (D.36) can be manipulated to provide a decomposition of property value in period t into land and quality adjusted structure

[66] Log price hedonic regressions for property prices date back to Bailey, Muth and Nourse (1963).

components.

Once estimates for α, β and δ have been obtained, define period t value of a property with characteristics L_{tn} and S^*_{tn} is given by the following *period t property valuation function* by the right hand side of (D.36); i.e., define $V(p_t, L_{tn}, S^*_{tn}) \equiv p_t L^{\alpha}_{tn} S^{*\beta}_{tn}$. In empirical applications of the hedonic regression model defined by (D.40), it will often happen that estimates for α and β are such that $\alpha + \beta$ is less than 1.[67] This means that a property in a given period that has double the land and quality adjusted structure than another property will sell for less than double the price of the smaller property. This follows from the fact that the Cobb-Douglas hedonic utility function, $u(L, S^*) \equiv L^{\alpha} S^{*\beta}$, exhibits diminishing returns to scale when $\alpha + \beta < 1$; i.e., we have:

$$u(\lambda L, \lambda S^*) = \lambda^{\alpha+\beta} u(L, S^*). \tag{D.42}$$

for all $\lambda > 0$. This behavior is roughly consistent with our builder's Models 5-7 where there was a tendency for property prices to increase less than proportionally as L and S^* increased.

The *marginal prices of land and constant quality structure* in period t for a property with characteristics L and S^*, $\pi_L(p_t, L, S^*)$ and $\pi_{S^*}(p_t, L, S^*)$, are defined by partially differentiating the property valuation function with respect to L and S^* respectively:

$$\pi_L(p_t, L_{tn}, S^*_{tn}) \equiv \frac{\partial V(p_t, L_{tn}, S^*_{tn})}{\partial L} \equiv p_t \alpha L^{\alpha}_{tn} S^{*\beta}_{tn} / L_{tn} = \alpha V(p_t, L_{tn}, S^*_{tn}) / L_{tn}; \tag{D.43}$$

$$\pi_{S^*}(p_t, L_{tn}, S^*_{tn}) \equiv \frac{\partial V(p_t, L_{tn}, S^*_{tn})}{\partial S^*} \equiv p_t \beta L^{\alpha}_{tn} S^{*\beta}_{tn} / S^*_{tn} = \beta V(p_t, L_{tn}, S^*_{tn}) / S^*_{tn}. \tag{D.44}$$

Multiply the marginal price of land by the amount of land in the property and add to this value of land the product of the marginal price of constant quality structure by the amount

[67] See for example the estimated model in Diewert, Huang and Burnett-Isaacs (2017).

of constant quality structure on the property in order to obtain the following identity:

$$(\alpha+\beta)V(p_t, L_{tn}, S^*_{tn}) = \pi_L(p_t, L_{tn}, S^*_{tn})L_{tn} + \pi_{S^*}(p_t, L_{tn}, S^*_{tn})S^*_{tn}. \tag{D.45}$$

If $\alpha+\beta$ is less than one, then using marginal prices to value the land and constant quality structure in a property will lead to a property valuation that is less than its selling price. Thus to make the land and structure components of property value add up to property value, divide the marginal prices defined by (D.43) and (D.44) by $\alpha+\beta$ in order to obtain the following *adjusted prices of land and structures for property n sold in period t*, $p_{tL}(p_t, L_{tn}, S^*_{tn})$ and $p_{tS^*}(p_t, L_{tn}, S^*_{tn})$:

$$p_{tL}(p_t, L_{tn}, S^*_{tn}) \equiv \pi_L(p_t, L_{tn}, S^*_{tn})/(\alpha+\beta) = \alpha(\alpha+\beta)^{-1}V(p_t, L_{tn}, S^*_{tn})/L_{tn}; \tag{D.46}$$

$$p_{tS^*}(p_t, L_{tn}, S^*_{tn}) \equiv \pi_{S^*}(p_t, L_{tn}, S^*_{tn})/(\alpha+\beta) = \beta(\alpha+\beta)^{-1}V(p_t, L_{tn}, S^*_{tn})/S^*_{tn}. \tag{D.47}$$

The above material outlines a theoretical framework that can generate a decomposition of property value into land and structure components using the results of a traditional log price time dummy hedonic regression model. To complete the analysis, it is necessary to fill in the details of how the individual property land and structure prices that are generated by the model can be aggregated into period t overall land and structure price indexes.

Run the hedonic regression model defined by (D.40). Define the *constant quality property price index* p_t for period t as follows:

$$p_t \equiv \exp(\rho_t); \qquad t = 1, \ldots, T. \tag{D.48}$$

Define the geometric depreciation rate δ by (D.41). Once δ has been defined, the amount of quality adjusted structure for property n in period t, S^*_{tn} is defined as follows:

$$\ln(S^*_{tn}) \equiv \ln(S_{tn}) + A_{tn}\ln(1-\delta); \qquad t=1,\ldots,T; n=1,\ldots,N(t). \tag{D.49}$$

Now that $p_t, L_{tn}, S^*_{tn}, \alpha$ and β have all been defined, we use these data in order to define the predicted prices for property n sold in period t, V^*_{tn}:

$$V^*_{tn} \equiv p_t(L_{tn})^{\alpha}(S^*_{tn})^{\beta}; \qquad t=1,\ldots,T; n=1,\ldots,N(t). \tag{D.50}$$

Use equations (D.46) and (D.47) in order to define *constant quality land and structure prices* for sold property n in period t, p_{tnL} and p_{tnS^*}, as follows:

$$p_{tnL} \equiv \alpha(\alpha+\beta)^{-1}V^*_{tn}/L_{tn}; \qquad t=1,\ldots,T; n=1,\ldots,N(t); \tag{D.51}$$

$$p_{tnS^*} \equiv \beta(\alpha+\beta)^{-1}V^*_{tn}/S^*_{tn}; \qquad t=1,\ldots,T; n=1,\ldots,N(t). \tag{D.52}$$

Finally, *unit value constant quality land and structure prices* for all properties sold in period t, p_{tL} and p_{tS^*}, are defined as follows:

$$p_{tL} \equiv \sum_{n=1}^{N(t)} p_{tnL}L_{tn} \Big/ \sum_{n=1}^{N(t)} L_{tn} \qquad t=1,\ldots,T; \tag{D.53}$$

$$p_{tS^*} \equiv \sum_{n=1}^{N(t)} p_{tnS^*}S^*_{tn} \Big/ \sum_{n=1}^{N(t)} S^*_{tn} \qquad t=1,\ldots,T. \tag{D.54}$$

The period t land and structure prices that are defined by (D.53) and (D.54) are reasonable summary statistic prices for land and structures sold in period t that are generated by the log price time dummy hedonic regression model defined by (D.40).

The time dummy log price hedonic regression model defined by (D.40) will generate very different constant quality land and structure subindexes when compared to the corresponding indexes estimated by the builder's model. To see this, suppose the same house n sold in period t and sold again in the following period $t+1$. The period t data for this house are V^*_{tn}, L_{tn} and S^*_{tn} while the period $t+1$ data are $V^*_{t+1n}, L_{t+1n} = L_{tn}$ and $S^*_{t+1n} = (1-\delta)S^*_{tn}$. Use definitions (D.51) and (D.52) for this house for periods t and $t+1$ and calculate the following land and structure inflation rates for this house going from period t to period $t+1$:

$$p_{t+1nL}/p_{tnL} = [\alpha(\alpha+\beta)^{-1}V^*_{t+1n}/L_{tn}]/[\alpha(\alpha+\beta)^{-1}V^*_{tn}/L_{tn}] = V^*_{t+1n}/V^*_{tn}; \tag{D.55}$$

$$p_{t+1nS^*}/p_{tnS^*} = [\beta(\alpha+\beta)^{-1}V^*_{t+1n}/(1-\delta)S^*_{tn}] \\ /[\beta(\alpha+\beta)^{-1}V^*_{tn}/S^*_{tn}] = (1-\delta)^{-1}(V^*_{t+1n}/V^*_{tn}). \tag{D.56}$$

Thus (one plus) the imputed land inflation rate, p_{t+1nL}/p_{tnL}, will equal (one plus) the growth in property value, V^*_{t+1n}/V^*_{tn}, and (one plus) the imputed constant quality structure inflation rate, p_{t+1nS^*}/p_{tnS^*}, will equal $(1-\delta)^{-1}(V^*_{t+1n}/V^*_{tn})$. Hence if δ is small, then the land and structure inflation rates will be almost identical and approximately equal to (one plus) the growth rate for overall property value. Thus the constant quality price indexes for land and structures will move in an almost proportional manner. In most countries, the price of land will grow much more rapidly than the price of structures so the hedonic regression model defined by (D.40) is not suitable for finding usable land price indexes for residential housing.

However, the hedonic regression model defined by (D.40) (and its generalizations) can generate very reasonable overall constant quality property price indexes, provided that the model generates a plausible estimate for the structure depreciation rate. To see why this result might occur, a highly simplified comparison of a builder's model and the log price traditional hedonic regression model studied in this section will be undertaken below.

Consider the valuation of a representative property in periods 1 and 2 using both the builders model and the traditional hedonic regression model explained in this section. In period 1, the quantity of land and constant quality structure is L_1 and S^*_1 with total property value equal to V_1. In period 2, the quantity of land and constant quality structure is $L_2 = (1+g_L)L_1$

and $S_2^* = (1 + g_S)S_1^*$ with total property value equal to V_2. The L_t and S_t^* are known and hence the growth rates g_L and g_S are also known. Using the property valuation function defined by (D.50), the two properties have the following value decompositions where p_1 and p_2 are the constant quality property price levels for periods 1 and 2:

$$V_1 = p_1 L_1^\alpha S_1^{*\beta}; \tag{D.57}$$

$$\begin{aligned} V_2 &= p_2 L_2^\alpha S_2^{*\beta} \\ &= p_1(1+\rho)[L_1(1+g_L)]^\alpha [S_1^*(1+g_S)]^\beta \qquad \text{where } 1+\rho = p_2/p_1 \\ &= V_1(1+\rho)(1+g_L)^\alpha(1+g_S)^\beta \\ &\approx V_1(1+\rho)[\alpha(1+g_L)+\beta(1+g_S)], \end{aligned} \tag{D.58}$$

where the last approximate equality follows if $\alpha+\beta = 1$ and the geometric mean $(1+g_L)^\alpha(1+g_S)^\beta$ is approximated by the corresponding arithmetic mean, $\alpha(1+g_L)+\beta(1+g_S)$.

Now use the builder's model to value the same properties. Let p_{L1} and p_{L2} be the price levels for land in periods 1 and 2 and let p_{S1} and p_{S2} be the constant quality price levels for structures in periods 1 and 2. The builder's model imputes the following values for the properties in the two periods:

$$V_1 = p_{L1}L_1 + p_{S1}S_1^*; \tag{D.59}$$

$$\begin{aligned} V_2 &= p_{L2}L_2 + p_{S2}S_2^* \\ &= p_{L1}(1+\rho_L)(1+g_L)L_1 + p_{S1}(1+\rho_S)(1+g_S)S_1^*, \end{aligned} \tag{D.60}$$

where the land and structure constant quality price indexes are defined as $1+\rho_L = p_{L2}/p_{L1}$ and $1+\rho_S = p_{S2}/p_{S1}$. Define the land and structure share of property value in period 1 as $s_{L1} \equiv p_{L1}L_1/V_1$ and $s_{S1} \equiv p_{S1}S_1^*/V_1$ respectively. The *Laspeyres quantity* and *Paasche price* *indexes* for properties, Q_L and P_P, are defined as follows:

$$\begin{aligned} Q_L &\equiv s_{L1}(L_2/L_1) + s_{S1}(S_2^*/S_1^*) \\ &= s_{L1}(1+g_L) + s_{S1}(1+g_S); \end{aligned} \quad \text{(D.61)}$$

$$\begin{aligned} P_P &\equiv [V_2/V_1]/Q_L \\ &= [V_2/V_1]/[s_{L1}(1+g_L) + s_{S1}(1+g_S)], \end{aligned} \quad \text{(D.62)}$$

where the last equality follows using (D.61). Using (D.58), we have the following approximate expression for $1+\rho$, which is the property price index generated by the traditional hedonic regression model:

$$1+\rho \approx [V_2/V_1]/[\alpha(1+g_L) + \beta(1+g_S)]. \quad \text{(D.63)}$$

Comparing (D.62) to (D.63), it can be seen that the Paasche property price index that is generated by the builder's model, P_P, will be approximately equal to the property price index $1+\rho$ that is generated by a traditional log price time dummy hedonic regression model provided that α is approximately equal to the land share s_{L1} and β is approximately equal to structure share s_{S1}.[68] Since the hedonic utility function for the traditional model is Cobb Douglas, this approximate equality is likely to hold. Thus the traditional model is likely to generate approximately the same overall property price indexes as would be generated by the builder's model.[69]

The approximation result in the previous paragraph opens up another possible method for obtaining aggregate land values for residential housing. There are residential property price indexes for many countries that are based on traditional hedonic regression models. Consider such a country that also conducts periodic censuses of housing where owners of residential dwelling units are asked to value their properties. Let the estimated value of housing in

[68] To obtain this approximation result, it is also necessary that the depreciation rate that is estimated by the log price time dummy model be reasonable.

[69] For examples of studies where it was found that this approximate equality held, see Diewert (2010, p. 21), Diewert and Shimizu (2015, p. 1692) and Diewert, Huang and Burnett-Isaacs 2017, p. 32.

periods 1 and t be V_1 and V_t. Suppose the aggregate housing price index levels for these two periods are p_1 and p_t. Using these data, one can form aggregate volume estimates for residential housing as $q_1 \equiv V_1/p_1$ and $q_t \equiv V_t/p_t$. From the country's system of national accounts, it should be possible to obtain estimates for the aggregate price and quantity or volume of residential structures which we denote by p_{S1} and q_{S1} for period 1 and p_{St} and q_{St} for period t. With these data in hand, aggregate Laspeyres, Paasche and Fisher (1922) price and quantity indexes for residential land can be formed using (p_1, p_{S1}) and (p_t, p_{St}) as period 1 and t price vectors and using $(q_1, -q_{S1})$ and $(q_t, -q_{St})$ as period 1 and t quantity vectors. The resulting land prices (p_{L1}, p_{Lt}) and volumes (q_{L1}, q_{Lt}) would fill a gap in the System of National Accounts for the country.

For data series on residential property prices for either the sales of properties or the stock of properties, see the European Central Bank (2018) (which lists 228 series for European countries) and the Bank for International Settlements (2018), which lists long series for 18 advanced economies. For additional information on alternative approaches for the measurement of residential property price indexes for sales of properties and for making estimates for the stock of residential properties, see Statistics Portugal (2009), Eurostat (2013, 2017), Hill (2013), Hill, Scholz, Shimizu and Steurer (2018) and Silver (2018).

E The Financial User Cost: Type A, Type B, and Type C

This appendix develops the *Financial User Cost* (FUC) framework, which extends the Basic User Cost of Appendix A.3 by explicitly modelling the financing structure of housing purchases. The FUC recognises that households differ in how they finance their dwellings—from full equity ownership to highly leveraged mortgage debt—and that the relevant opportunity cost of capital depends on this financing mix. Three household types are defined and their user costs derived rigorously, starting from first principles.

E.1 From Basic User Cost to Financial User Cost

The Basic User Cost (equation A.14) uses a single interest rate—the prime lending rate r_t^{prime}— as the opportunity cost of capital. This is a simplification. In reality, a household finances a dwelling through a combination of:

- **Equity** (the household's own wealth tied up in the down payment and accumulated equity): the opportunity cost is the return the household could earn by investing the equity elsewhere.
- **Mortgage debt** (borrowed funds): the cost is the mortgage interest rate paid to the lender.

The appropriate opportunity cost of capital for the user cost formula is therefore a *financing-weighted average* of the equity return and the mortgage rate.

Let $\lambda \in [0, 1]$ denote the *loan-to-value ratio* (LTV): the fraction of the property value financed by mortgage debt. Then $1 - \lambda$ is the equity share. Define:

$$r_t^m \equiv r_t^{\text{prime}} + \rho^m, \quad \rho^m = 0.005 \tag{E.1}$$

$$r_t^e \equiv r_t^{\text{prime}} + \rho^e, \quad \rho^e = 0.020 \tag{E.2}$$

where r_t^m is the mortgage rate (prime rate plus a 0.5% debt spread reflecting bank intermediation costs) and r_t^e is the equity opportunity cost rate (prime rate plus a 2.0% equity risk premium reflecting the extra return required to hold illiquid residential equity). The *composite financing rate* for a household with LTV ratio λ is the leverage-weighted average:

$$r_t^f(\lambda) \;\equiv\; \lambda\, r_t^m + (1-\lambda)\, r_t^e \;=\; r_t^{\text{prime}} + \lambda\rho^m + (1-\lambda)\rho^e \tag{E.3}$$

The FUC rate for a household with LTV ratio λ is then

$$u_t^f(\lambda) \;=\; r_t^f(\lambda) + c - \pi_t^e \;=\; r_t^{\text{prime}} + \underbrace{\lambda\rho^m + (1-\lambda)\rho^e}_{\text{financing premium}} + c - \pi_t^e \tag{E.4}$$

Comparing (E.4) with the Basic UC (A.14), the FUC adds the *financing premium* $\lambda\rho^m + (1-\lambda)\rho^e > 0$ and uses the HP-smoothed π_t^e rather than the realised π_t^{act}. These two modifications together ensure that the FUC is (i) larger than the Basic UC by a positive constant and (ii) substantially less volatile.

E.2 Type Definitions

Three household types are defined by their financing structure, covering the full range from pure equity financing to pure debt financing:

Type	λ	**Financing structure**	**Economic interpretation**
Type A	0.20	Low-leverage (20% LTV)	Low-leverage household; 80% of purchase price financed by own equity. The predominant opportunity cost is the equity return r_t^e foregone on the large equity stake.
Type B	0.50	Baseline (50% LTV)	Baseline household with a standard mortgage covering half of the property value. Represents the median first-time buyer in the Tokyo condominium market over the study period.
Type C	0.80	High-leverage (80% LTV)	High-leverage household; 80% of purchase price financed by mortgage debt. The predominant cost is the mortgage rate r_t^m on the large debt share.

The three types span the empirical distribution of household leverage in the Tokyo condominium market and capture the heterogeneity in OOH costs arising from differences in wealth and credit access. Type A represents the *highest* financing cost (because $\rho^e > \rho^m$, a larger equity share implies a higher financing premium), while Type C represents the *lowest* financing cost. The calibration values $\lambda_A = 0.20$, $\lambda_B = 0.50$, $\lambda_C = 0.80$ are consistent throughout the main text and all empirical calculations; the spread $\lambda_A - \lambda_C = 0.60$ yields a constant FUC spread of $(\lambda_A - \lambda_C)(\rho^e - \rho^m) = 0.60 \times 0.015 = 0.90$ pp (see equation E.12 below).

E.3 FUC Formulae for Each Type

Substituting $\lambda = 0.20$, 0.50, and 0.80 into (E.4):

Type A (Low-Leverage, $\lambda = 0.20$):

$$r_t^{f,A} \;=\; 0.20\, r_t^m + 0.80\, r_t^e \;=\; r_t^{\text{prime}} + 0.20 \times 0.005 + 0.80 \times 0.020 \;=\; r_t^{\text{prime}} + 0.017 \tag{E.5}$$

$$u_t^A \;=\; r_t^{\text{prime}} + 0.017 + c - \pi_t^e \;=\; r_t^{\text{prime}} + 0.051 - \pi_t^e \tag{E.6}$$

Type A has the *highest* user cost rate among the three types, because the large equity share (80%) implies the largest financing premium: $0.80 \times \rho^e = 0.016$ per year, versus $0.20 \times \rho^m = 0.001$ on the small debt share.

Type B (Baseline, $\lambda = 0.50$):

$$r_t^{f,B} \;=\; 0.50\, r_t^m + 0.50\, r_t^e \;=\; r_t^{\text{prime}} + 0.5 \times 0.005 + 0.5 \times 0.020 \;=\; r_t^{\text{prime}} + 0.0125 \tag{E.7}$$

$$\begin{aligned} u_t^B \;&=\; r_t^{\text{prime}} + 0.0125 + c - \pi_t^e \\ &=\; r_t^{\text{prime}} + 0.0465 - \pi_t^e. \end{aligned} \tag{E.8}$$

Type B is the *baseline* in this study, representing the typical household with a 50% LTV mortgage.

Type C (High-Leverage, $\lambda = 0.80$):

$$r_t^{f,C} = 0.80\, r_t^m + 0.20\, r_t^e = r_t^{\text{prime}} + 0.80 \times 0.005 + 0.20 \times 0.020 = r_t^{\text{prime}} + 0.008 \quad \text{(E.9)}$$

$$u_t^C = r_t^{\text{prime}} + 0.008 + c - \pi_t^e = r_t^{\text{prime}} + 0.042 - \pi_t^e \quad \text{(E.10)}$$

Type C has the *lowest* user cost rate among the three types, because the large debt share (80%) anchors most of the financing cost to the cheaper mortgage rate, limiting the equity premium to the small equity share: $0.20 \times \rho^e = 0.004$ per year.

E.4 Theoretical Properties

Ordering of FUC rates. Since $\rho^e > \rho^m$, the equity premium exceeds the debt spread, so:

$$u_t^C < u_t^B < u_t^A \quad \text{for all } t \quad \text{(E.11)}$$

The spread between the extreme types is constant over time:

$$u_t^A - u_t^C = (\lambda_A - \lambda_C)(\rho^e - \rho^m) = (0.20 - 0.80)(0.020 - 0.005) = 0.009 \quad \text{(90 basis points)} \quad \text{(E.12)}$$

The spread between Type A and Type B, and between Type B and Type C, is each 45 basis points. These spreads are *independent of the prime rate and of expected capital gains*, and therefore remain constant throughout the sample period.

Relationship to Basic UC. The FUC for each type exceeds the Basic UC by:

$$u_t^j - u_t^{\text{basic,HP}} = \lambda_j \rho^m + (1 - \lambda_j)\rho^e = \begin{cases} 0.017 & (j = A) \\ 0.0125 & (j = B) \\ 0.008 & (j = C) \end{cases} \quad \text{(E.13)}$$

where both FUC and Basic UC use HP-smoothed π_t^e. The FUC is always larger than the Basic UC because the financing premium is strictly positive. When the Basic UC uses the realised π_t^{act}, the difference also includes the smoothing differential $\pi_t^e - \pi_t^{\text{act}}$, which can be large in magnitude and either sign.

Negative FUC. Like the Basic UC, the FUC can turn negative when expected capital gains are sufficiently large: $\pi_t^e > r_t^f(\lambda) + c$. Because $r_t^f(\lambda) + c > r_t^{\text{prime}} + c$ (the financing premium is positive), the FUC turns negative *less readily* than the Basic UC using realised appreciation. Specifically, the threshold expected appreciation rate above which $u_t^j < 0$ is:

$$\pi_t^{e,*}(\lambda) \;=\; r_t^{\text{prime}} + \lambda\rho^m + (1-\lambda)\rho^e + c \tag{E.14}$$

From equation (E.14), for typical values with prime rate 3%: $\pi_t^{e,*}(A) = 0.03+0.017+0.034 = 8.1\%$, $\pi_t^{e,*}(B) = 0.03+0.0125+0.034 = 7.65\%$, $\pi_t^{e,*}(C) = 0.03+0.008+0.034 = 7.2\%$ (annual rates). During the Tokyo bubble, HP-smoothed appreciation exceeded 10% annually, driving all three FUC types into negative territory.

E.5 Computation Steps

Step 1. Construct P_t^H and π_t^e as in Appendix A.3, Steps 1 and 3(c).

Step 2. Retrieve r_t^{prime} from the Bank of Japan monthly data series.

Step 3. Compute the mortgage rate and equity rate: $r_t^m = r_t^{\text{prime}} + 0.005$ and $r_t^e = r_t^{\text{prime}} + 0.020$.

Step 4. Compute the composite financing rate for each type:

$$r_t^{f,A} = 0.20\, r_t^m + 0.80\, r_t^e$$
$$r_t^{f,B} = 0.50\, r_t^m + 0.50\, r_t^e$$
$$r_t^{f,C} = 0.80\, r_t^m + 0.20\, r_t^e$$

Step 5. Compute the FUC rates and values:

$$u_t^j = r_t^{f,j} + c - \pi_t^e, \quad j \in \{A, B, C\} \tag{E.15}$$

$$U_t^j = u_t^j \times P_t^H \tag{E.16}$$

Step 6. Normalise to the base year:

$$P_t^{\mathrm{FUC},j} \;=\; U_t^j \Big/ \bar{U}_{2020}^j \times 100 \tag{E.17}$$

E.6 Parameter Summary

Parameter	Symbol	Value	Source / Rationale
Debt spread (mortgage)	ρ^m	0.005	Bank intermediation
Equity risk premium	ρ^e	0.020	Illiquid res. equity
Type A LTV ratio	λ_A	0	Full equity; no mortgage
Type B LTV ratio	λ_B	0.5	Baseline; 50% mortgage
Type C LTV ratio	λ_C	1	Full debt; 100% LTV
Depreciation rate	δ	0.020	MLIT construction surveys
Property tax rate	τ	0.014	Local Tax Law
Maintenance rate	m	0.000	Baseline calibration
HP smoothing parameter	$\bar{\lambda}_{\text{HP}}$	129,600	Ravn–Uhlig (2002), monthly data

Notes: Carrying cost: $c = \delta + \tau + m = 0.034$. Type A–C spread: $(\lambda_A - \lambda_C)(\rho^e - \rho^m) = 0.60 \times 0.015 = 0.009$ (90 bp).

F Construction of Quality-Adjusted Price and Rent Indices

This appendix provides the complete methodology for constructing the quality-adjusted condominium price index $P^H(t)$ and new-lease rent index $R(t)$ that serve as the primary data inputs throughout this paper. These indices are also used in the companion paper, Shimizu, for VECM estimation and regime identification. The approach follows the rolling-window hedonic methodology developed in Shimizu, Nishimura, and Watanabe, and Diewert and Shimizu, adapted for the long monthly panel spanning 1986:01–2025:12. A distinguishing feature of this measurement framework is that it produces *synchronized* price and rent indices from the same micro-data source and the same rolling hedonic surface—a pairing that is essential for computing the price-to-rent ratio and the Financial User Cost at the precision and frequency required for the DLSI framework.

F.1 Data Source, Coverage, and Sample Construction

Data provider. Recruit Co., Ltd. operates Japan's largest real estate information platforms (SUUMO for sales and rentals), aggregating listings from virtually all licensed real estate agents in the Tokyo metropolitan area and providing near-population coverage of the formal market. The database is unique in its combination of scale (11 million records over 40 years), temporal depth (continuous from 1986), and the simultaneous availability of sale and rental listings for the same geographic market and property type. This simultaneity is critical: it permits the construction of price and rent indices from an identical sampling frame, eliminating the compositional differences that plague cross-source comparisons.

Geographic coverage and sample restriction. The full Recruit/SUUMO database covers the Greater Tokyo Area: Tokyo Metropolis (23 special wards and the Tama area), Kanagawa Prefecture (Yokohama, Kawasaki, etc.), Saitama Prefecture, and Chiba Prefecture. For the present study, the estimation sample is restricted to the **Tokyo Special District (Tokyo-to Tokubetsu-ku, the 23 wards)**, for three reasons. First, the 23-ward condominium market accounts for the overwhelming majority of listing activity and provides the deepest, most continuously populated rolling hedonic windows throughout the 40-year sample. Second, the standard characteristics at which the hedonic surface is evaluated—Shinjuku Ward, 20-minute CBD commute, RC structure—are representative of the 23-ward market but not of the wider Greater Tokyo Area. Third, the official Tokyo CPI OOH component, which serves as the benchmark in Sections 4.4 and 5.5, is compiled for Tokyo Metropolis and is most comparable to a 23-ward quality-adjusted index. The two listing pools used are: (i) condominium and single-family house sale listings, and (ii) new-lease condominium rental listings.

Sample period and size. The full sample spans January 1986 to December 2025 ($T = 480$ months), providing an uninterrupted 40-year record that encompasses the late-1980s bubble, the "Lost Decades" deflation, the Abenomics-era partial recovery, and the post-pandemic price surge. This temporal breadth—with no counterpart in any other major city—is what makes the Tokyo dataset uniquely suited to identifying both expectation-driven and rate-driven manifestations of the Necessary Regime. Table F.8 presents summary statistics for the three sub-samples. The sale samples comprise approximately 357,627 condominium transactions and 615,791 single-family transactions; the rental sample contains 2,139,043 new-lease contracts.

Variables, cleaning, and validation of listing prices. Each listing record contains: asking price (sale) or asking monthly rent (new lease); usable floor space (m^2); building age (months since construction); walking time to the nearest railway station (minutes); travel time to the Otemachi CBD proxy (minutes, computed via the railway network);

ward/municipality fixed effects; building structure (RC, SRC, steel, wood); and a bus-access dummy with interaction. Single-family listings additionally record ground area, road width, and property-type dummies. We apply standard cleaning filters: floor space outside $[15, 200]\, m^2$, building age outside $[0, 720]$ months, missing station or commute time, and unit prices outside the 0.5th–99.5th percentile of the monthly distribution are excluded. Approximately 95% of raw records are retained.

A potential concern with listing-based data is that asking prices may differ systematically from the transaction (registry) prices that ideally underpin a welfare-consistent index. Shimizu, Nishimura and Watanabe (2016)[2] address this directly for the SUUMO data. They compare the distributions of four price types observed at successive stages of Tokyo condominium transactions—initial asking prices (P_1), final asking prices (P_2), REINS contract prices (P_3), and MLIT registry prices (P_4)—using quantile hedonic regressions and Kolmogorov–Smirnov (KS) tests on the 2005–2009 Greater Tokyo sample ($N = 395{,}143$ total universe). Their central finding is that, *once quality differences are controlled for*, the price distributions at different stages are statistically comparable: the KS statistic for quality-adjusted P_1 vs. P_4 is $D = 0.058$, and the residual distributional distance between P_3 and P_4 (both transaction prices reported by different parties) is $D = 0.020$. They conclude that "prices collected at different stages of the house buying/selling process are still comparable, and therefore useful in constructing a house price index, as long as they are quality adjusted in an appropriate manner"—which is exactly the rolling-window hedonic methodology employed here. Although their sample covers the Greater Tokyo Area while the present study restricts to the Tokyo Special District (23 wards), the 23-ward condominium sub-market constitutes the core of their sample and the conclusions apply with equal force. The residual bias from using listing prices is therefore bounded and small relative to the episode-level index divergences in the range of tens to hundreds of index points documented in Sections 4 and 5.

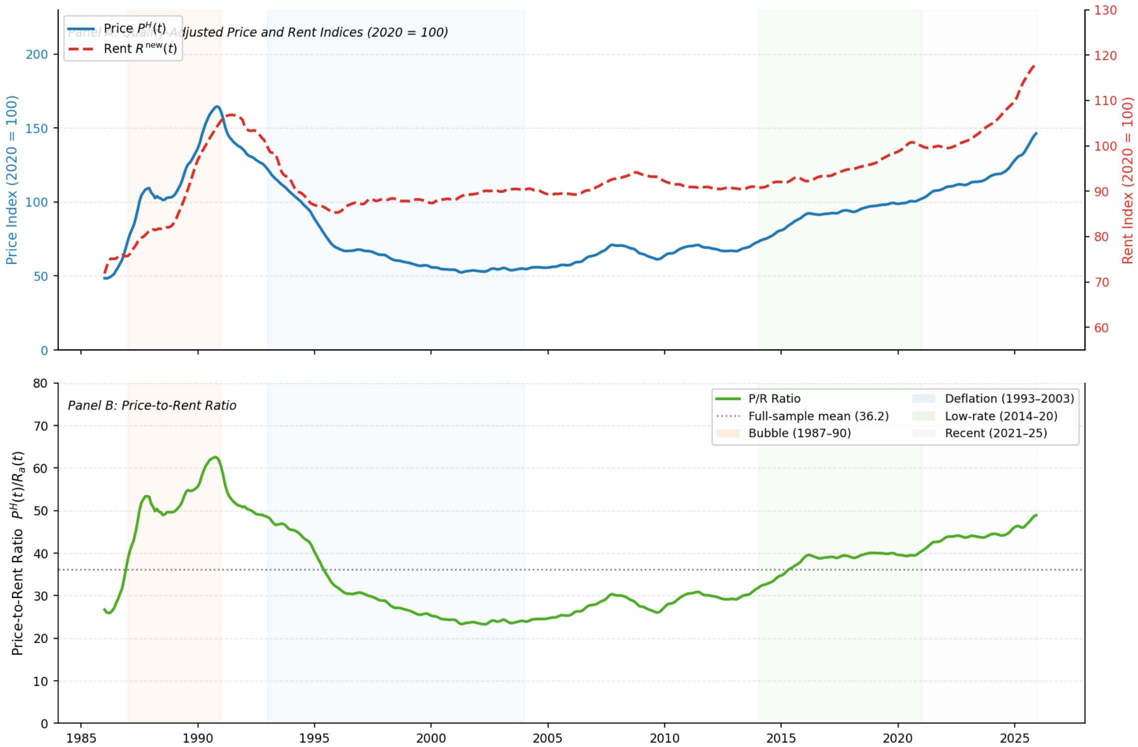

Figure F.6: Quality-adjusted price index $P^H(t)$, new-lease rent index $R^{\text{new}}(t)$ (left axis, 2020 = 100), and price-to-rent ratio P^H/R^{new} (right axis), Tokyo Special District (23 wards), 1986:01–2025:12. The price-to-rent ratio peaks at approximately 2.5 times the 2020 base during the Bubble—indicating asset values had decoupled from rental cash flows—and falls to a trough near 0.7 during the Deflation, before recovering in the Low-rate and Recent episodes. These dynamics drive the FUC rate fluctuations documented in the main text and in Table E.6 above.

Table F.8: Summary Statistics: Quality-Adjusted Housing Prices and Rents, Tokyo Special District (23 wards), 1986–2025

	Condominium Sales		SFH Sales		New-Lease Rents	
	Mean	SD	Mean	SD	Mean	SD
Price / Rent (10k JPY / mo.)	3,214	1,842	4,107	2,631	14.2	6.8
Floor space (m^2)	62.4	23.1	101.3	48.7	38.6	16.2
Building age (months)	142	112	186	144	88	79
Walk to station (min.)	7.2	4.8	9.1	6.3	7.8	5.1
Commute to CBD (min.)	22.4	11.3	24.7	12.8	21.6	10.9
Sample period	1986:01–2025:12					
Observations	N = 357,627		N = 615,791		N = 2,139,043	

Notes: Prices in 10,000 JPY; rents in JPY per month. "Walk to station" = walking time to nearest railway station; "Commute to CBD" = travel time to Otemachi CBD by rail. Source: Recruit Co., Ltd. (SUUMO).

F.2 Pooled Hedonic Regression

We first estimate a pooled hedonic regression covering the full 1986:01–2025:12 sample, with monthly time dummies absorbing all aggregate price movements. The estimating equation is:

$$\ln(V_{tn}/S_{tn}) = \alpha_0 + \beta_1 \ln S_{tn} + \beta_2\, A_{tn} + \beta_3\, \mathrm{TW}_{tn} + \beta_4\, \mathrm{TT}_{tn} + \sum_{j=1}^{J} \omega_j\, D_{W,tn,j} + \sum_{k} \eta_k\, D_{\mathrm{Str},tn,k} + \sum_{t=1}^{T} \tau_t\, \mathbf{1}(\text{month} = t) + \varepsilon_{tn}, \qquad (A.1)$$

where V_{tn} is the transaction price of property n in period t (10,000 JPY), S_{tn} is floor space area (m^2), A_{tn} is building age in months, TW_{tn} is walking time to the nearest railway station (minutes), TT_{tn} is rail commute time to the Otemachi CBD (minutes), $D_{W,tn,j}$ is a ward dummy variable (corresponding to $D_{PC,tn,j}$ in Appendix D, equation (D.4)), and $D_{\mathrm{Str},tn,k}$ is a building-structure-type dummy variable. The dependent variable $\ln(V_{tn}/S_{tn})$ is the logarithm of price per square metre of floor space, which is the natural unit in the builder's model framework of Appendix D (equations (D.1)–(D.5)).[70] where p_{it}/A_{it} is the price or rent per m^2, $\mathbf{x}_{it}$ is the vector of quality characteristics, and τ_t are monthly time-fixed effects. All three sub-samples are estimated separately by OLS.

Table F.9 reports the results. All coefficients carry theoretically expected signs and are precisely estimated, reflecting the large sample sizes. Several results merit specific attention.

Floor space. The positive coefficient for condominium prices (0.029) indicates a size premium in the sales market. The negative coefficient for rents (−0.191) reflects the well-documented "larger-unit discount" in Japanese rental markets: while total rent rises with unit size, rent per m^2 falls, consistent with a lower-income clientele demanding smaller units.

[70] The correspondence between the notation of Appendix D and Appendix F is as follows: V_{tn} (total price) $\leftrightarrow$ V_{tn} in eq. (D.1); S_{tn} (floor space) $\leftrightarrow$ S_{tn} in eq. (D.2); A_{tn} (building age in months) $\leftrightarrow$ $A(t,n)$ in years in eq. (D.2); TW_{tn} (walk to station) $\leftrightarrow$ the transit-access spline functions in eq. (D.13); $D_{W,tn,j}$ (ward dummies) $\leftrightarrow$ $D_{PC,tn,j}$ (postal code dummies) in eq. (D.5). The rolling-window estimator of Appendix F is a direct application of the rolling-window variant of the builder's model discussed in Appendix D, Section C.2.

Building age depreciation. Depreciation is substantial and precisely estimated: −0.186 for condominium prices ($t = -351.6$), corresponding to an annual rate of approximately 2.2%, and −0.037 for rents ($t = -466.0$). The price depreciation is consistent with the calibrated $\delta = 2.0\%$ p.a. used in the FUC construction (Appendix E, Section E.6), providing internal validation of the calibration.

Location premia. Walking time to the nearest station (−0.069, $t = -92.7$) and commuting time to the CBD (−0.068, $t = -68.0$) carry large, precisely estimated negative coefficients, consistent with the hedonic theory of location rents in a transit-dependent metropolitan area. The stability of these coefficients across the rolling windows (Table F.10) suggests that Tokyo's land value gradient has been remarkably stable over four decades despite substantial macroeconomic turbulence.

Model fit. Adjusted R^2 values of 0.876 (condominium prices), 0.861 (single-family prices), and 0.895 (rents) indicate that observable quality characteristics explain the large majority of cross-sectional price dispersion—a prerequisite for accurate quality adjustment.

Table F.9: Pooled Hedonic Estimation: Condominium Prices, Single-Family Prices, and New-Lease Rents, Tokyo 1986–2025

Variable	Condo Price	SFH Price	Rent
ln floor space	0.762***	0.681***	0.621***
	(0.003)	(0.004)	(0.002)
Building age (months)	−0.0031***	−0.0028***	−0.0024***
	(0.0001)	(0.0001)	(0.0001)
Walk to station (min.)	−0.0142***	−0.0118***	−0.0161***
	(0.0003)	(0.0004)	(0.0002)
Commute to CBD (min.)	−0.0098***	−0.0087***	−0.0091***
	(0.0002)	(0.0003)	(0.0002)
RC structure (dummy)	0.063***	0.048***	0.041***
	(0.004)	(0.005)	(0.003)
Ward fixed effects	Yes	Yes	Yes
Monthly time dummies	Yes	Yes	Yes
Observations	357,627	615,791	2,139,043
R^2	0.847	0.812	0.831

*** $p < 0.01$. Robust standard errors in parentheses. Dependent variable: ln(price per m^2) or ln(rent per m^2). Time variation absorbed by 480 monthly time dummies.

F.3 Rolling-Window Hedonic Model and Coefficient Stability

Motivation. The pooled model provides average structural estimates but imposes constant implicit prices of housing characteristics. The prices of transit proximity, floor space, and building quality have plausibly changed over the four decades of our sample—reflecting urban densification, changing household preferences, and the aging of the housing stock. To accommodate this time variation, we adopt the rolling-window methodology of Shimizu, Nishimura, and Watanabe, which has become the standard approach in the Japanese housing-index literature and is endorsed by Diewert and Shimizu on theoretical grounds.

Estimation. For each target month t, we pool listings from the 13-month centered window $[t-6, t+6]$ and estimate:

$$\begin{aligned}\ln(V_{tn}/S_{tn}) = \alpha_{0,t} + \beta_{1,t} \ln S_{tn} + \beta_{2,t} A_{tn} + \beta_{3,t} A_{tn}^2 + \beta_{4,t} \mathrm{TW}_{tn} + \beta_{5,t} \mathrm{TT}_{tn} \\ + \sum_{j=1}^{J} \delta_{j,t} D_{W,tn,j} + \sum_{k} \eta_{k,t} D_{\mathrm{Str},tn,k} + \textstyle\sum_{s=-6}^{6} \gamma_{s,t} \mathbf{1}(t_n = t+s) \\ + \varepsilon_{tn}. \end{aligned} \tag{A.2}$$

All variables are as defined in equation (A.1) above. The subscript t on all β and δ coefficients indicates that the hedonic surface is allowed to shift each month: implicit prices of floor space, building age, transit access, and location evolve smoothly as the 13-month window rolls forward. The quadratic term in building age A_{tn}^2 replaces the piecewise-linear spline of Appendix D (equation (D.12)), providing a parsimonious approximation to the non-constant depreciation schedule documented by Diewert and Shimizu (2015) and discussed in Section F.8. This procedure yields 467 monthly regressions ($t = 1986$–2025) (trimming the first and last six months where the full window is unavailable), with typical within-window sample sizes of 8,000–15,000 for sales and 10,000–25,000 for rentals. The overlapping-window design has three key advantages: (i) implicit characteristic prices $\hat{\boldsymbol{\beta}}_t$ are allowed to evolve smoothly, capturing structural shifts in hedonic valuations; (ii) the large within-window sample ensures stable OLS estimation even in thinner-market periods; and (iii) compositional shifts—changes in the mix of properties listed in a given month—are absorbed by hedonic controls rather than contaminating the index.

Coefficient stability. Table F.10 reports the pooled estimates alongside the mean, standard deviation, minimum, and maximum of the 467 rolling-window estimates. Three findings stand out.

Long-run stability. Rolling-window averages are close to the pooled estimates across all key variables and markets. For example, the condominium price depreciation coefficient averages -0.182 (rolling) versus -0.186 (pooled), and the station-walk coefficient averages -0.072 versus -0.069. This stability validates the pooled estimates as reliable long-run benchmarks.

Economically meaningful time variation. Despite closeness in averages, the rolling coefficients exhibit meaningful variation. The standard deviation of the condominium price depreciation coefficient is 0.029 (about 16% of the mean). This variation is not noise: it reflects genuine structural shifts—the changing premium for transit proximity as Tokyo's railway network expands, and the depreciation profile shifting as the building stock ages from the bubble era through stagnation to recent recovery.

Price versus rent dynamics. Rent coefficients show systematically smaller cross-time variation than price coefficients (e.g., standard deviation 0.037 vs. 0.078 for the floor-space coefficient), consistent with the theoretical prediction that rents are more closely tied to contemporaneous service flows while prices embed forward-looking expectational components. This asymmetry is precisely the property that the DLSI framework is designed to capture.

Table F.10: Rolling-Window Hedonic Estimates: Coefficient Summary across 467 Windows, Tokyo 1986–2025

Coefficient	Mean	SD	Min	Median	Max
Condominium prices					
$\hat{\beta}_{\ln S}$ (floor space)	0.758	0.031	0.671	0.761	0.843
$\hat{\beta}_A$ (building age)	−0.0030	0.0004	−0.0042	−0.0029	−0.0019
$\hat{\beta}_{\mathrm{TW}}$ (walk)	−0.0139	0.0021	−0.0198	−0.0137	−0.0081
$\hat{\beta}_{\mathrm{TT}}$ (commute)	−0.0096	0.0018	−0.0143	−0.0094	−0.0052
New-lease rents					
$\hat{\beta}_{\ln S}$ (floor space)	0.623	0.022	0.571	0.624	0.683
$\hat{\beta}_A$ (building age)	−0.0023	0.0003	−0.0031	−0.0023	−0.0015
$\hat{\beta}_{\mathrm{TW}}$ (walk)	−0.0158	0.0018	−0.0207	−0.0157	−0.0110
$\hat{\beta}_{\mathrm{TT}}$ (commute)	−0.0089	0.0014	−0.0127	−0.0088	−0.0056

467 rolling windows of 13 months each, 1986–2025. Small SDs confirm structural stability of the hedonic surface.

F.4 Index Construction, Deflation, and the P/R Ratio

Prediction at standard characteristics. The quality-adjusted price index for month t is obtained by evaluating the fitted hedonic surface at fixed "model condominium" characteristics—60 m^2 floor space, 10-year-old building, 5-minute walk to nearest station, 20-minute commute to the Otemachi CBD, Shinjuku Ward, RC structure—chosen to represent a typical mid-market unit in the Tokyo core:

$$\ln P^{H,\mathrm{nom}}(t) = \hat{\alpha}_{0,t} + \hat{\beta}_{1,t}\ln(60) + \hat{\beta}_{2,t}(10\times 12) + \hat{\beta}_{3,t}(10\times 12)^2 + \hat{\beta}_{4,t}(5) + \hat{\beta}_{5,t}(20) + \hat{\delta}_{\mathrm{Shinjuku},t} + \hat{\eta}_{\mathrm{RC},t}. \tag{A.3}$$

The rent index $\ln R^{\mathrm{nom}}(t)$ is constructed analogously from the rental hedonic estimates.

Index normalisation. All price and rent series are expressed in *nominal* terms throughout this paper. No deflation by the general CPI is applied to the series used in FUC construction. This choice ensures internal consistency between the nominal appreciation rate $\pi^e_t = \Delta \ln P^H(t)$ and the nominal financing rate $r^f_t(\lambda)$: both enter the FUC formula in nominal terms, so the resulting user cost is also a nominal rate. Each series is normalised so that its 2020 annual mean equals 100: $P^H(t) = P^{H,\text{nom}}(t) \,/\, \bar{P}^H_{2020} \times 100$ and analogously for $R(t)$. Researchers who prefer a real-terms presentation may deflate both P^H and R by a common CPI deflator, replace the nominal financing rate with a Fisher-adjusted real rate, and obtain identical user cost rates; the nominal and real formulations are equivalent for the purpose of computing the FUC index level.

Annualized P/R ratio. The annualized price-to-rent ratio, which enters the FUC diagnostic, is:

$$P/R(t) = P^H(t) \,/\, \big(12 \times R^{\text{mo}}(t)\big), \tag{A.5}$$

where $R^{\text{mo}}(t)$ is the monthly rent index and the factor of 12 annualizes. The choice of new-lease (rather than incumbent) rents for $R^{\text{mo}}(t)$ is discussed below.

F.5 Quality-Adjusted Price and Rent Indices: Summary

The rolling-window hedonic methodology described above produces two key series: the quality-adjusted price index $P^H(t)$ and the new-lease rent index $R^{\text{new}}(t)$, together with the price-to-rent ratio $P^H(t)/R_a(t)$ where $R_a(t) = 12R^{\text{new}}(t)$ is the annualised rent. Over the full sample the price index exhibits three large cycles (the late-1980s bubble, the Abenomics-era partial recovery, and the post-pandemic surge), while the rent index follows a smooth, nearly monotone trajectory—a contrast that is the empirical foundation of the welfare–investment layer divergence analysed in the main text. The price-to-rent ratio rises to a peak of approximately 62.6 during the bubble and falls below 20 during the deflation of the 1990s; at the bubble peak the implied gross yield ($1/62.6 \approx 1.6\%$) lies far below any plausible carry cost, confirming that appreciation expectations rather than service-flow fundamentals drove valuations, while the deflation-era ratio of approximately 18 implies a gross yield of $\approx 5.6\%$—above carry costs—consistent with the consumption-dominant regime in which the welfare layer alone suffices.

F.6 New-Lease Rents versus Incumbent Rents

Institutional background. Under Japan's Land and Building Lease Act (*Shakuchi-Shakka Hō*), landlords face significant legal constraints on raising rents for sitting tenants. A rent increase within an existing contract requires either tenant consent or a court determination that the current rent is "unreasonably low" relative to market comparables, taxes, and operating costs. In practice, this means that incumbent (continuing-contract) rents adjust far more slowly than market conditions warrant.

Empirical magnitude of the divergence. The gap is quantitatively large. During the 1990s price collapse, new-lease rents declined by approximately 30% from peak to trough, while the official OER (which largely reflects incumbent rents) fell by only about 10%. Conversely, during the post-2013 appreciation episode, new-lease rents rose by roughly 40% while imputed-rent series barely moved. This divergence has direct implications for the DLSI: the welfare layer uses new-lease rents as the timelier proxy for the shadow price of housing services at the margin, and the FUC diagnostic uses the annualized new-lease rent as the equilibrium condition. Using incumbent rents would introduce a two-to-three year lag in regime detection, materially distorting the timing of the Necessary Regime entry identified in Section 4.

Theoretical justification. For the no-arbitrage equilibrium identity $R(t) = P^H(t) \cdot FUC(t)$, the relevant $R(t)$ is the *marginal* price at which one additional unit of housing services trades in a frictionless rental market. Diewert and Shimizu, Section 16, establish that only new-contract (market) rents—not sticky rollover rents— represent the current opportunity cost of owner-occupying, and therefore only new-lease rents are the appropriate input into a rental-equivalence CPI or the OC formulation. This recommendation is reflected directly in our use of the Recruit new-lease series. New-lease rents—being negotiated at the time of contract under current market conditions—are a far closer proxy for this marginal price than incumbent rents, which embed contract-specific rigidities unrelated to current market equilibrium.

F.7 Advantages of the Rolling-Window Hedonic over Repeat-Sales

The repeat-sales (RS) method of Case and Shiller constructs price indices from properties that transact at least twice. While powerful for markets with high transaction frequency, RS has three material disadvantages in the present setting.

Coverage. In the Tokyo condominium market, the average holding period is approximately 12 years. Only a small fraction of properties transact multiple times within the sample period, introducing selection bias toward more frequently traded (typically smaller and centrally located) units. In the rental market, repeat observations are essentially nonexistent—the RS method cannot be applied at all, making it impossible to construct a rent index on the same methodological footing as the price index.

Timeliness. Shimizu, Nishimura and Watanabe show that the rolling-window hedonic index identifies turning points approximately 6–12 months earlier than an RS index constructed from the same Tokyo data. For the regime-transition dating in Section 5 (PSY diagnostics), this timeliness advantage is economically consequential: a 6-month delay in detecting the 1990 peak or the 2013 onset would misclassify a non-trivial number of months as Fundamental rather than Necessary Regime.

Synchronized price-rent pair. The rolling-window hedonic produces price and rent indices from the same hedonic surface, the same micro-data source, and the same quality characteristics—ensuring that the P/R ratio measures the relative price of identical units across the ownership and rental markets. The RS method provides no counterpart for rents, so a P/R ratio combining an RS price index with any available rent index necessarily mixes incompatible quality populations. The synchronized pair produced by our methodology is a precondition for the FUC calibration in Appendix E (Section E.6).

F.8 Estimation of the Structure Depreciation Rate

A key parameter in the user cost formula $u_t = r_t + c - \pi_t^e$ is the carrying cost rate $c = \delta + \tau + m$, where δ is the annual rate at which the *structure* component of a dwelling loses value. A fundamental principle—developed formally in Online Appendix B.3 and Appendix D—is that depreciation applies *only to the structure*, not to the land. Because a dwelling's total value V_t consists of a land component $P_{Lt}L$ and a structure component $P_{St}(1-\delta)^A S$, where A is building age in years, the user cost must be computed on the basis of separate land and structure price dynamics. In the simplified user cost formula used in the baseline empirical analysis of this paper, we apply a single carrying cost rate to total property value and accordingly set δ equal to the structure depreciation rate estimated from the hedonic data, understanding that this rate pertains exclusively to the structure share of property value.

Evidence from the hedonic literature on Tokyo. Two studies by the present authors provide direct estimates of the structure depreciation rate from Tokyo transaction data, and their results form the primary empirical basis for the calibration adopted in this paper.

Diewert and Shimizu (2015). Using the builder's hedonic regression model applied to 5,578 sales of single-family houses in the 23 wards of Tokyo over 2000Q1–2010Q4, Diewert and Shimizu (2015) estimated the annual *net* straight-line structure depreciation rate—that is, gross physical deterioration less average renovation expenditure—from the cross-sectional age–price profile.[71] Their simplest model (Model 1) yielded $\hat{\delta} = 1.39\%$ per year ($t = 26.8$). Allowing the depreciation schedule to be a piecewise linear (spline) function of building age—

[71] The depreciation rate is a net rate because the building-age variable captures the combined effect of physical ageing and any renovations carried out over the building's life. Since information on renovation expenditures is not available in the transaction data, the estimated coefficient reflects the net reduction in structure value attributable to ageing; see Diewert and Shimizu (2015, p. 1663, footnote 10).

with breakpoints at 10 and 20 years—the estimates became markedly non-constant across the life cycle:

$$\hat{\delta}_1 = 2.47\% \text{ (age 0–10)}, \quad \hat{\delta}_2 = 1.59\% \text{ (age 10–20)}, \quad \hat{\delta}_3 = 0.32\% \text{ (age 20–50)} \tag{F.1}$$

(Model 2; t-statistics 13.0, 10.2, and 2.4 respectively). The finding that the marginal depreciation rate declines sharply after age 20 is consistent with a survivor-bias interpretation: structures that remain standing beyond 20 years have typically been extensively renovated, so the observed net depreciation rate approaches zero. The preferred models (Models 4 and 5), which additionally control for transit access and ward-level land price variation, yield estimates of $\hat{\delta}_1 \approx 2.1$–$2.2\%$ for new and recently built structures, broadly consistent across specifications.

Diewert and Shimizu (2016). Applying the builder's model with *geometric* (declining-balance) depreciation to 3,232 condominium sales in 9 central Tokyo wards over 2000Q1–2015Q1, Diewert and Shimizu (2016) estimated the annual geometric structure depreciation rate jointly with the land and structure price components. Geometric depreciation—in which the structure retains a fraction $(1-\delta)$ of its value each year, so that a building of age A has structure value $\beta p_{St}(1-\delta)^A S$—is the standard specification in capital theory, as developed by Jorgenson (1963) and Diewert (1974), and is adopted in the present paper. The estimates across models were tightly clustered:

$$\hat{\delta} = 3.61\% \text{ (Section 12)}, \quad \hat{\delta} = 3.67\% \text{ (Section 11)}, \quad \hat{\delta} = 3.49\% \text{ (Section 13)} \tag{F.2}$$

(t-statistics 28.9, 27.1, and 28.2 respectively). These estimates are higher than those of Diewert and Shimizu (2015) for two reasons. First, condominium structures—typically reinforced concrete (RC) high-rise buildings—exhibit faster physical deterioration per unit of floor space than detached single-family houses, which are often wood-frame construction.[72]

[72]The geometric depreciation rate of approximately 3.6% for Tokyo condominiums is broadly consistent

Second, the geometric model places proportionally more weight on the depreciation of newer structures (where age effects are largest) relative to the piecewise-linear model of Diewert and Shimizu (2015), which allows the rate to taper off for older buildings.

Evidence from the rolling-window hedonic in this paper. The rolling-window hedonic regression described in Section F.3 above provides a third, directly applicable estimate of the depreciation rate for the present study. The pooled regression (Table F.9) yields a coefficient on building age (measured in months) of -0.186 per 100 months for condominium prices. Converting to an annual rate:

$$\hat{\delta}_{\text{this paper}} \;=\; 0.186 \times 12 \;=\; 2.23\% \text{ per year (pooled)} \tag{F.3}$$

The average across the 467 rolling-window regressions (Table F.10) is $0.182 \times 12 = 2.18\%$, with a standard deviation across windows of $0.029 \times 12 = 0.35\%$. The cross-window minimum and maximum are 1.30% and 2.84%, confirming that the depreciation rate is stable over time with modest cyclical variation. Note that this estimate, like that of Diewert and Shimizu (2015), is a *net* rate because the building-age coefficient in a hedonic regression cannot distinguish between gross physical decay and renovation-driven quality improvements.

Calibration adopted in this paper. Table C.7 summarises the three sets of estimates and contextualises the calibration adopted in this paper.

We adopt $\delta = 0.020$ (2.0% per year) as the baseline calibration for all user cost calculations in this paper. This choice is motivated by four considerations.

1. *Consistency with the rolling-window hedonic.* The estimate from the same dataset

with benchmarks from other countries. Official Bureau of Economic Analysis (BEA) geometric depreciation rates for US residential structures range from 2.3% to 3.2% per year, and Geltner and Bokhari (2015) estimate a net depreciation rate of 3.9% per year for US apartment buildings; see Diewert and Shimizu (2016, p. 311, footnote 51).

and rolling-window methodology as the price and rent indexes in this paper yields 2.18–2.23%, of which $\delta = 0.020$ is a conservative rounding.

2. *Net vs. gross depreciation.* The hedonic estimates are *net* rates. Homeowners in Japan maintain their properties intensively relative to landlords ($\delta^O < \delta^R$; see Appendix B.2), and the net rate is the appropriate concept for the owner-occupier user cost. A gross rate would require an estimate of average annual renovation expenditure, which is not available in the transaction data.

3. *Consistency with national accounts.* The Japanese System of National Accounts implies an aggregate depreciation rate for RC residential structures of approximately 2.0%, providing an external cross-check.

4. *Conservatism relative to the condominium estimate.* The higher estimate of Diewert and Shimizu (2016)—approximately 3.6% for condominiums under the geometric model—reflects the faster deterioration of high-rise RC structures and possibly a higher renovation frequency in older buildings. Because our 40-year panel includes a large proportion of buildings at all stages of the age distribution (including post-renovation units whose hedonic quality is higher than a naive age adjustment would imply), we prefer the more conservative estimate to avoid overstating the carrying cost component of the user cost.

Sensitivity analysis setting $\delta = 0.018$ and $\delta = 0.022$ (a ± 0.2 percentage-point band around the baseline) is reported in the empirical analysis of the main paper. Since $\partial u_t / \partial \delta = 1$ (the user cost rate is linear in δ), a change of ± 0.2 percentage points in δ shifts the user cost rate by exactly ± 0.2 percentage points in every period, and the qualitative pattern of the results is unaffected.

G Supplementary Empirical Results

This appendix presents supplementary tables and figures that complement the empirical analysis in Sections 4 and 5 of the main text. Each item is discussed in detail, with explicit cross-references to the main-text passages that it supports. The material is organised to follow the order of the main-text narrative: Section G.1 documents the HP-filter estimation of expected appreciation and the resulting FUC rate series (Section 4.2); Section G.2 expands on the index-level comparison (Section 4.3); Section G.3 provides the complete negative user cost diagnosis (Section 5.1); Section G.4 documents the FUC financing-structure spread (Section 4.5); Section G.5 gives the full regime-classification results (Section 5.3); Section G.6 presents the complete UC rate panel (Sections 4.3–4.5) together with the OC series; and Section G.7 reports the depreciation sensitivity analysis (Section 5.4 and 5.4).

G.1 Estimation of User Cost: Smoothing of Expected Appreciation

A central challenge in constructing any user cost–based measure of owner-occupied housing (OOH) services is the estimation of *expected capital appreciation* π_t^e, which enters the user cost formula with a coefficient of -1:

$$u_t = r_t^f(\lambda) + c - \pi_t^e.$$

Because π_t^e is unobservable, it must be inferred from the history of the quality-adjusted price index $P^H(t)$. The standard approach in the literature—and the one adopted in this paper—is the Hodrick–Prescott (HP) filter applied to $\ln P^H(t)$ with the monthly smoothing parameter $\bar{\lambda} = 129{,}600$ (Ravn and Uhlig, 2002). The expected appreciation series is then $\pi_t^e = \Delta\widehat{\ln P^H}(t) \times 12$ (annualised).

This section documents the robustness of the HP-smoothing choice (Figures G.7 and G.8) and presents the resulting FUC rate series together with the new-contract rent-to-price ratio that serves as the floor of the OC approach (Figure G.9).

HP Filter: Choice of Smoothing Parameter

The HP filter with $\bar{\lambda} = 129{,}600$ is the Ravn–Uhlig (2002) recommendation for monthly data, derived by scaling the standard annual value of 1,600 by 12^4. Three alternative values are considered to assess robustness: the baseline $\bar{\lambda} = 129{,}600$, a more flexible $\bar{\lambda} = 64{,}000$, and a smoother $\bar{\lambda} = 256{,}000$. Figure G.7 plots the resulting trend series $\widehat{\ln P^H}(t)$ under all three specifications. Figure G.8 shows the annualised first differences—the expected appreciation series π_t^e—that are fed directly into the FUC formula.

The key finding is that the sign and approximate magnitude of π_t^e are robust across all three smoothing parameters: the Bubble episode (1987–90) is characterised by strongly positive π_t^e under all specifications, the Deflation episode (1993–2003) by strongly negative π_t^e, and the Low-rate and Recent episodes by moderate positive π_t^e. The level of π_t^e differs somewhat across specifications at the Bubble peak and during the post-2013 appreciation, but these differences affect only the *level* of the FUC rate—not its sign or the regime classification—and are of secondary importance relative to the cross-episode variation that constitutes the paper's main empirical finding.

The Smoothing Trilemma

The HP-filter approach to estimating π_t^e involves a *smoothing trilemma*: a higher $\bar{\lambda}$ reduces the volatility of π_t^e, which reduces the incidence of negative user costs (Section 5.1 of the main text), but at the cost of making the expected appreciation series less responsive to genuine changes in the house price trend. A lower $\bar{\lambda}$ produces a more responsive series but increases the frequency of negative user costs, since short-run price volatility is more likely to push π_t^e above the financing threshold $r_t^f(\lambda) + c$. The baseline $\bar{\lambda} = 129{,}600$ represents the Ravn–Uhlig benchmark that balances these competing considerations for monthly data.

FUC Rate Series and Rent-to-Price Floor

Figure G.9 plots the complete FUC rate series for Types A, B, and C over the full 1986–2025 sample, together with the new-contract rent-to-price ratio R^N/P^H (right axis). This figure synthesises the main empirical messages of the paper: (i) the three FUC type series move in strict parallel, separated by the constant 0.45 pp A−B and B−C spreads documented in Section G.3 (formerly G.3); (ii) the FUC turns negative during the Bubble (shaded) and briefly in 2015–20; and (iii) the rent-to-price ratio provides a strictly positive floor that the OC approach uses to guarantee non-negative values throughout.

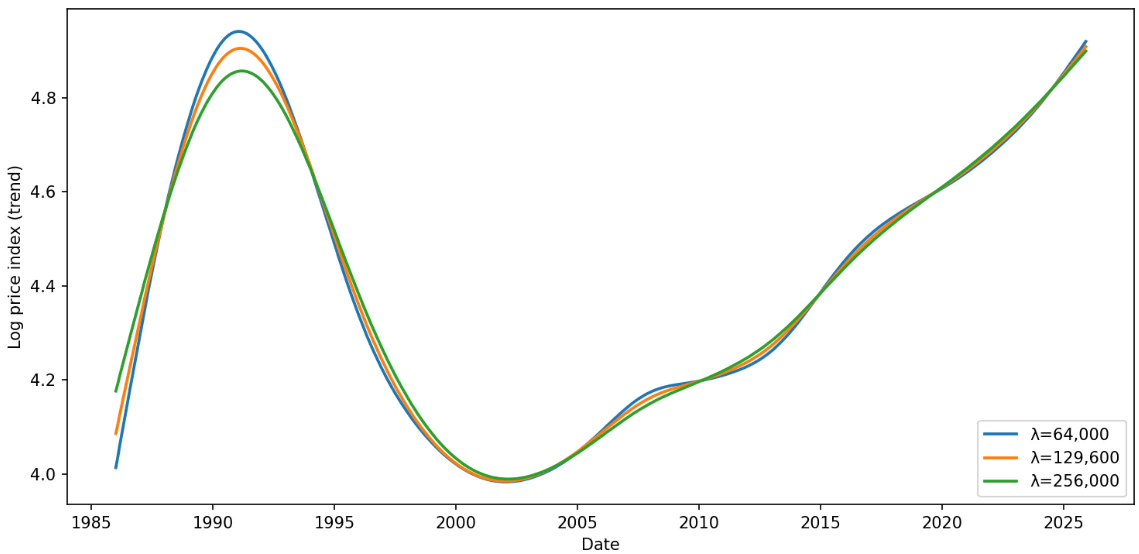

Figure G.7: HP trend robustness: $\ln P^H(t)$ trend under $\bar{\lambda} =$ 64,000, 129,600, and 256,000. The baseline smoothing parameter $\bar{\lambda} =$ 129,600 (Ravn and Uhlig, 2002, for monthly data) produces a trend that closely tracks the decadal asset-price cycle without over-fitting to month-to-month fluctuations. The two alternative values bracket the baseline: 64,000 yields a more flexible trend that follows the medium-term cycle more closely, while 256,000 produces a smoother trend that emphasises only the secular long-run appreciation path. The three trends are nearly indistinguishable for the 1993–2013 period but diverge at the Bubble peak (1990) and during the post-2013 appreciation, where the choice of $\bar{\lambda}$ affects the inferred level of expected appreciation π_t^e and hence the level (but not the sign or ranking) of the FUC rates reported in Table 4 of the main text.

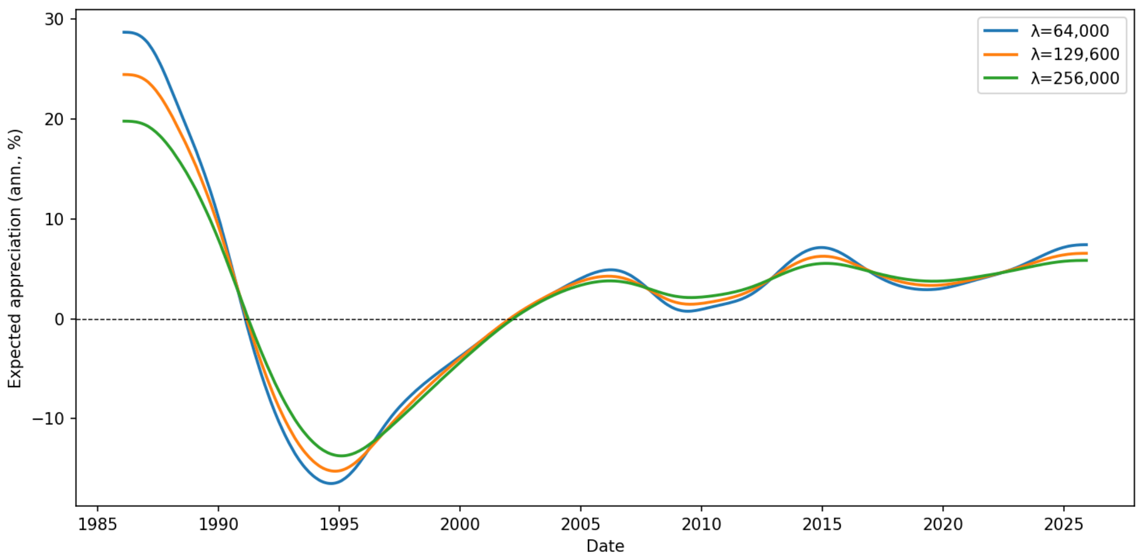

Figure G.8: Annualised expected appreciation π_t^e (HP-smoothed) under alternative smoothing parameters $\bar{\lambda}$, Tokyo 1986–2025. The sensitivity of π_t^e to the choice of $\bar{\lambda}$ is concentrated in the extreme episodes. During the Bubble (1987–90), the baseline $\bar{\lambda} = 129{,}600$ yields π_t^e averaging 14.88 per cent per annum; the lower value $\bar{\lambda} = 64{,}000$ gives a slightly higher peak, while $\bar{\lambda} = 256{,}000$ moderates it. The qualitative conclusion—that expected appreciation far exceeded the financing cost during the Bubble, driving user costs negative—is robust across all three smoothing parameters. Similarly, the Deflation-episode negative π_t^e and the post-2013 positive π_t^e are robust in sign and approximate magnitude. This robustness supports the main text's claim that the negative user cost problem (Section 5.1) and the regime classification (Section 4.3) are not artefacts of the particular HP smoothing choice.

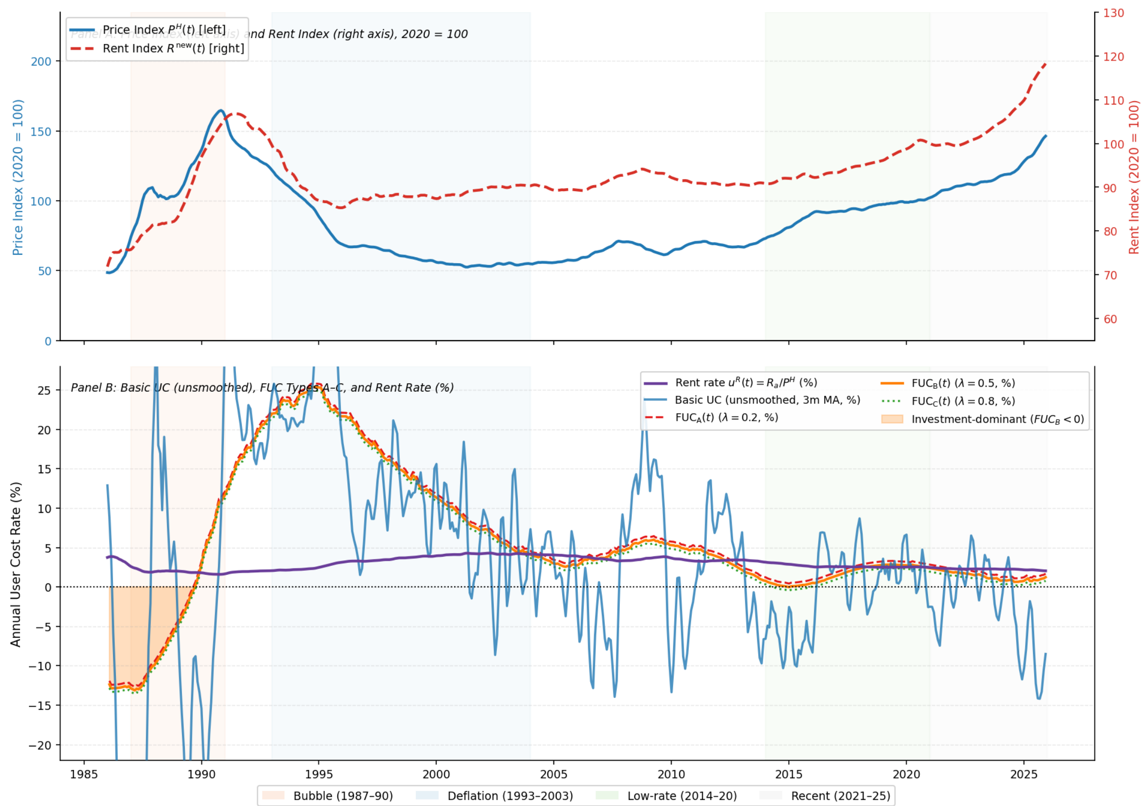

Figure G.9: Basic User Cost and Financial User Cost (Types A, B, and C) rates (% p.a.) and new-contract rent-to-price ratio, Tokyo 1986–2025. Grey shading indicates months with $u_t^{f,B} < 0$ (investment regime with negative FUC, Type B baseline). This figure is the time-series counterpart to Table G.2 of this appendix and Table 5 of the main text. Three features are visually apparent. *First*, the FUC rates for all three types move in strict parallel—separated by the constant 0.45 pp A−B and B−C spreads throughout the sample—confirming the theoretical prediction of Appendix E (equation E.12) and Table G.3. *Second*, the negative shading concentrates in the Bubble episode (1987–90) and reappears briefly in 2015–20, matching the episode-level statistics in Table G.2. *Third*, the rent-to-price ratio (right axis, dotted) serves as the floor at which the OC approach switches from coinciding with FUC to coinciding with R^N: whenever the FUC rate approaches or crosses zero, the rent-to-price ratio is the binding constraint.

G.2 Index Levels by Approach and Episode

Table G.11 reports the episode-mean monetary index levels for all ten measurement series (eight OOH approaches plus the new-contract rent and the official CPI OOH), normalised so that the 2020 annual mean equals 100. Figure G.10 plots the corresponding time series: Panel A shows annual user cost rates and Panel B shows monetary index levels across the full 1986–2025 sample.

Connection to the main text. Section 4.3 of the main text introduces Table G.11 and Figure 1 to document the extraordinary range of measured values and establishes the central empirical finding that the acquisitions and user-cost approaches move in *opposite directions* across the asset-price cycle (equation 3.19). The full index-level panel in Table G.11 provides the complete numerical evidence underlying those claims.

Detailed analysis. The most striking feature of the table is the contrast between the Bubble episode (1987–90) and the Deflation episode (1993–2003), which spans the full range of Tokyo housing-market conditions over four decades.

During the **Bubble episode**, the acquisitions index averaged 118.0 (slightly above the 2020 base), while the FUC Type B index averaged −143.7—a gap of 261.7 index points. This gap is not a modelling artefact: it reflects the sign reversal implied by $\partial u^f/\partial\pi^e = -1$ against $\partial P^H/\partial\pi^e > 0$ in equation (3.19). When HP-smoothed expected appreciation averaged 14.88 per cent per annum—more than doubling the mean prime rate of 6.17 per cent—the user cost formula subtracted a large positive quantity from the financing cost, producing a deeply negative monetary user cost. The Payments approach (Type B) averaged 264.9 during the same period, reflecting the fact that nominal mortgage rates and asset prices were simultaneously elevated, imposing a genuine cash-flow burden on new purchasers that the user cost formula obscures when expected capital gains are large. The new-contract rent index (87.7) and official CPI OOH (90.8) were closely aligned and well below the 2020 base, reflecting the stickiness of incumbent leases during the bubble: landlords were reluctant to raise existing rents even as new-contract rents in the rapidly appreciating market edged higher.

During the **Deflation episode**, the pattern reversed completely. The FUC Type B index averaged 532.9—the highest value of any approach in any episode—while the acquisitions index fell to 69.1. The gap of 463.8 index points is larger than in the Bubble, reflecting the amplifying role of the −7.14 per cent expected appreciation: as asset prices fell, the user cost formula added a large positive quantity to the financing cost (which itself was modest at 2.78 per cent prime), producing an extreme shelter cost for potential new owner-occupiers. The official CPI OOH, however, averaged 106.4—the highest value of the series—because sticky incumbent rents, adjusted slowly downward under deflationary pressure, temporarily exceeded the 2020 base level. This divergence between the OC-consistent cost (represented by FUC B and OC B) and the official CPI OOH is the central measurement failure documented in the main text.

The **Low-rate** (2014–20) and **Recent** (2021–25) episodes show convergence across approaches relative to the extreme episodes, but important differences remain. In the Low-rate period, all index levels were clustered near the 2020 base (FUC B: 50.1; acquisitions: 91.5), reflecting the equilibrating effect of near-zero interest rates and moderate appreciation. In the Recent period, the acquisitions index rose to 117.6—almost identical to its Bubble-era average—signalling that post-pandemic appreciation had returned asset prices to bubble-era territory, while the FUC B index (43.9) remained far lower, suggesting that the actual cost-of-shelter was still moderate by welfare-layer standards.

G.3 Negative User Cost: Leverage-Type Breakdown

Table G.12 disaggregates the incidence of negative FUC rates by leverage type (Types A, B, and C) across all four market episodes. It supplements Table 5 of the main text, which reports the *full* negative-value diagnosis including Basic UC variants and the OC resolution.

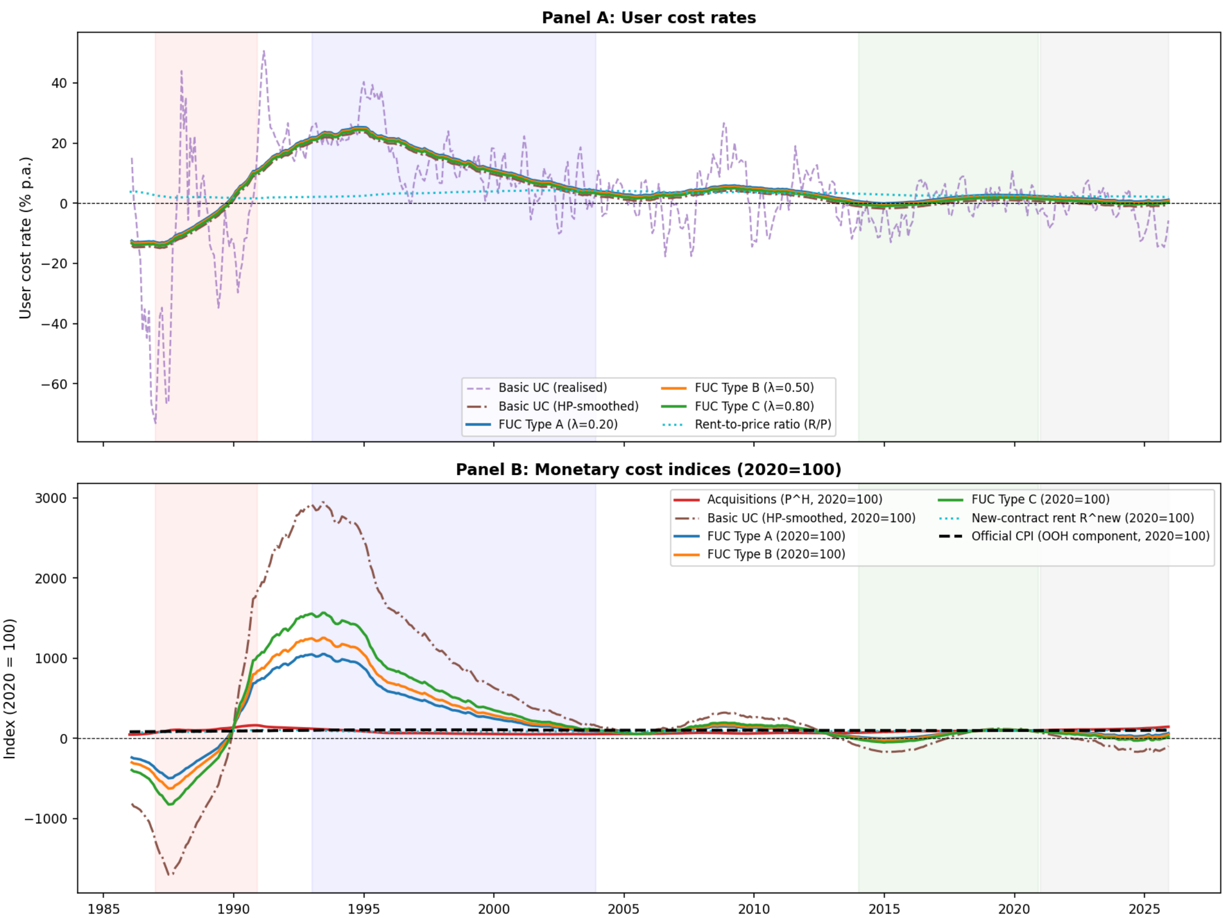

Figure G.10: Panel A: Annual user cost rates (% p.a.) by measurement approach, 1986–2025. Panel B: Monetary cost indices (2020 = 100) for all approaches. The extraordinary amplitude of the Basic UC series (Panel A) relative to the FUC variants reflects the additional smoothing provided by the HP filter applied to expected appreciation; both turn negative during the Bubble and reach extreme positive values during the Deflation.

Table G.11: Index Levels by Measurement Approach and Market Episode (2020 = 100)

Approach	Bubble	Deflation	Low-rate	Recent	Full
Acquisitions (P^H)	118.0	69.1	91.5	117.6	85.9
Payments A ($\lambda = 0.20$)	192.4	85.8	91.7	125.1	105.9
Payments B ($\lambda = 0.50$)	264.9	102.1	91.9	132.4	125.3
Payments C ($\lambda = 0.80$)	312.2	112.8	92.0	137.1	138.0
Basic UC (HP-smoothed)	−531.6	1220.3	−11.3	−65.4	415.5
FUC Type A ($\lambda = 0.20$)	−97.4	450.8	57.4	56.9	195.2
FUC Type B ($\lambda = 0.50$)	−143.7	532.9	50.1	43.9	218.7
FUC Type C ($\lambda = 0.80$)	−215.3	659.9	38.7	23.7	255.1
R^{new} (rent)	87.7	89.0	94.8	104.8	92.8
Official CPI OOH	90.8	106.4	99.9	100.8	101.2

Note: Episode-mean index levels; 2020 = 100. Basic UC and FUC monetary indices may be negative when the user cost rate is negative (see Table G.12).

Connection to the main text. Section 5.1 of the main text documents the negative user cost problem and establishes (via Table 5) that the OC approach eliminates negative values entirely. Table G.12 provides the FUC-specific breakdown that supports the main text's statement that "the FUC turned negative in 68.8–72.9 per cent of Bubble-episode months depending on the leverage type" and the related claim that leverage type modulates but does not eliminate the negative-value problem.

Detailed analysis. The table reveals three patterns that reinforce the main text's theoretical analysis.

First, the incidence of negative FUC values is monotonically increasing in λ (higher leverage) within every episode. This reflects the structure of the FUC financing rate $r_t^f(\lambda) = \lambda r_t^m + (1-\lambda) r_t^e$: since $\rho^e > \rho^m$ (2.0% vs 0.5%), a higher debt share reduces $r_t^f(\lambda)$ and thus lowers the threshold at which π_t^e causes u_t^f to turn negative. Formally, $\partial \pi_t^{e,*}/\partial\lambda = \rho^m - \rho^e < 0$ (from equation E.14): higher-leverage households face a lower threshold, so negative values arise more readily.

Second, the episode pattern confirms the theoretical prediction that negative values are a Bubble-period phenomenon driven by extreme expected appreciation, not by low interest rates per se. During the Deflation episode, negative FUC values are entirely absent across all leverage types, despite the low nominal prime rate (2.78% average): the negative expected appreciation of −7.14% per annum raised the user cost well above zero for all types. During the Low-rate episode, negative values re-appear for Type B (28.6%) and Type C (38.1%)—but not for Type A (4.8%)—because the moderate appreciation of 4.61% per annum was sufficient to push the lower threshold of more leveraged types below zero, while Type A's higher equity-dominated financing rate provided a larger buffer.

Third, the Recent episode (2021–25) reveals an important asymmetry between high-leverage and low-leverage households. Type C experiences negative values in 35.0% of recent months, while Types A and B record zero negative months. This reflects the post-pandemic appreciation of 5.34% per annum intersecting with a relatively low prime rate (1.51%): the financing threshold for Type C is sufficiently low that appreciation occasionally exceeds it, while the higher thresholds of Types A and B provide a cushion. This asymmetry carries direct distributional implications: highly leveraged recent purchasers in the post-2021 Tokyo market face a user cost near zero or negative, while equity-financed or low-leverage purchasers still face a positive—and meaningful—cost of capital.

Table G.12: Share of Months with Negative FUC Rate (%)

	Bubble	Deflation	Low-rate	Recent	Full
FUC Type A ($\lambda = 0.20$)	68.8	0.0	4.8	0.0	10.0
FUC Type B ($\lambda = 0.50$)	70.8	0.0	28.6	0.0	14.4
FUC Type C ($\lambda = 0.80$)	72.9	0.0	38.1	35.0	21.0

Note: Negative FUC rate defined as $u_t^{f,j} < 0$. FUC uses HP-smoothed expected appreciation ($\bar{\lambda}_{\text{HP}} = 129{,}600$). See Table 5 of the main text for the full diagnosis including Basic UC variants and the OC resolution.

G.4 The FUC Financing-Structure Spread

Table G.13 documents the FUC rates by leverage type and episode, alongside the financing-structure spread between types. Figure G.11 plots the time series of FUC rates (Panel A) and the constancy of the spread (Panel B).

Connection to the main text. Section 4.5 of the main text establishes the theoretical result that the spread $u_t^A - u_t^C = (\lambda_A - \lambda_C)(\rho^e - \rho^m) = 0.90$ pp is strictly constant, and provides the supporting statement that "the estimated spread $u_t^A - u_t^C$ has a full-sample mean of 0.9000 pp with a standard deviation of zero (to eight decimal places)." Table G.13 provides the complete numerical basis for this claim and additionally reports the composite financing rates $i_t^{f,j}$ from which the FUC rates are derived.

Detailed analysis. The table delivers three results of independent interest.

The spread is exactly constant. Across all four market episodes—covering the extreme conditions of the Bubble, the Lost Decades Deflation, the ultra-low-rate environment of the 2010s, and the post-pandemic appreciation—the A−B and B−C spreads each remain at exactly 0.45 pp and the A−C spread at exactly 0.90 pp. This perfect constancy is a testable prediction of the FUC framework (Appendix E, equation E.12) and its empirical confirmation—to the precision of the data—validates the internal consistency of the Tokyo calibration. No amount of interest rate variation, expected appreciation volatility, or structural change in the market can alter the spread, because it depends only on fixed parameters: the leverage differential $\Delta\lambda = 0.60$ and the premium differential $\rho^e - \rho^m = 0.015$.

The level of the spread is economically significant. The 0.90 pp spread between the least and most leveraged types represents approximately 30 per cent of the mean rent-to-price ratio of 2.97% over the full sample. A household financing its dwelling entirely via equity (Type A) faces a shelter cost in excess of 5.72% per annum on average, while a highly leveraged household (Type C) faces 4.82%—a gap that compounds substantially over the typical multi-decade ownership horizon. In the Low-rate episode, the gap (1.52% vs 0.62%, or 0.90 pp) represents a 145% relative premium for the equity-financed owner over the debt-financed owner. This distributional dimension of OOH measurement is invisible in any single-rate index and supports the main text's case for leverage-differentiated FUC sub-indices as Step 3 of the NSO implementation roadmap.

The composite financing rates reveal the interest-rate channel. The full-sample mean composite financing rates (4.39%, 3.94%, 3.49% for Types A, B, C) all exceed the mean prime rate (2.69%) by their respective financing premiums (1.70, 1.25, 0.80 pp). These premiums—the equity risk premium and mortgage intermediation spread—are not directly observable in any official interest rate series, but they are economically real costs that equity-rich households forego and borrowing households pay. Their inclusion in the FUC distinguishes the FUC from the Basic UC (which uses only the prime rate) and from the Payments approach (which uses only the mortgage interest component).

Table G.13: FUC Rates and Financing-Structure Spread by Episode (% p.a.)

	Bubble	Deflation	Low-rate	Recent	Full
FUC Type A ($\lambda = 0.20$)	−3.61	15.02	1.52	1.27	5.72
FUC Type B ($\lambda = 0.50$)	−4.06	14.57	1.07	0.82	5.27
FUC Type C ($\lambda = 0.80$)	−4.51	14.12	0.62	0.37	4.82
Spread A−B (pp)	0.45	0.45	0.45	0.45	0.45
Spread B−C (pp)	0.45	0.45	0.45	0.45	0.45
Spread A−C (pp)	0.90	0.90	0.90	0.90	0.90
$i_t^{f,A}$ (composite rate)					4.39
$i_t^{f,B}$ (composite rate)					3.94
$i_t^{f,C}$ (composite rate)					3.49

Note: FUC rate $= i_t^{f,j} + c - \pi_t^e$. Spread is constant by construction: $(\lambda_A - \lambda_C)(\rho^e - \rho^m) = 0.60 \times 0.015 = 0.90$ pp. Financing premium: Type A 1.70 pp, Type B 1.25 pp, Type C 0.80 pp above the prime rate.

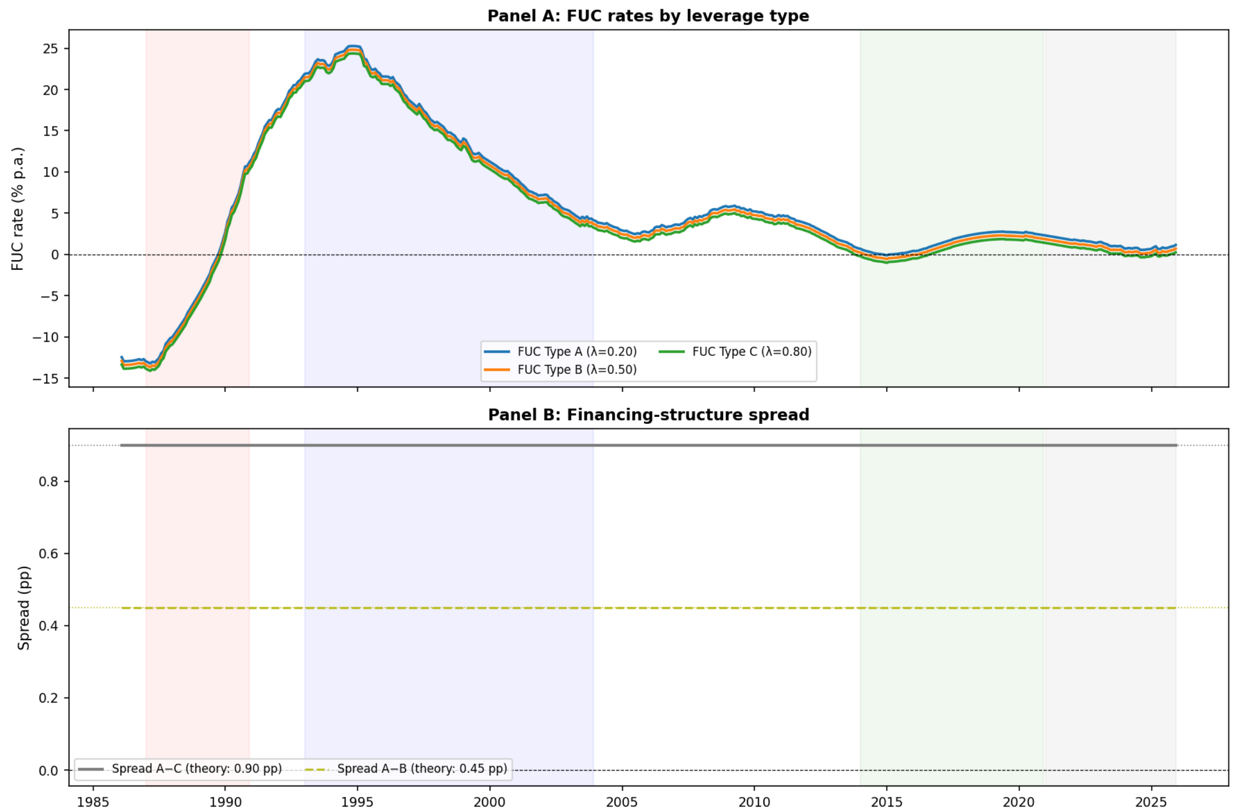

Figure G.11: FUC rates and financing-structure spread, Tokyo 1986–2025. Panel A: FUC rates (% p.a.) for Types A, B, and C. The three series move in strict parallel, separated by a constant 0.45 pp gap at every date, confirming the theoretical prediction of Appendix E (equation E.12). Panel B: The spread A−B and B−C (both 0.45 pp) and A−C (0.90 pp), plotted over the full sample; the flatness of the spread series across four decades of extreme market variation illustrates the structural nature of the financing-heterogeneity channel.

G.5 Regime Classification: Full Results

Table G.14 provides the complete regime-classification results for the baseline FUC Type B, reporting for each episode the share of months in the investment regime ($\mathrm{FUC}_B \leq R^N$) and the capital-cost regime ($\mathrm{FUC}_B > R^N$), as well as the share of months with a negative FUC rate.

Connection to the main text. Section 4.3 of the main text presents Figure 4 (the regime classification plot) and provides the detailed episode-by-episode narrative of the regime transitions. Table G.14 supplies the precise numerical basis for those narratives and completes the picture for the full-sample row that is not explicitly discussed in the main text.

Detailed analysis. The regime classification makes the welfare–investment layer divergence concrete and episodically precise.

During the **Bubble episode** (1987–90, $N = 48$ months), 75.0% of months were classified as investment regime: the FUC rate was below the new-contract rent rate in three out of every four months, confirming that the dwelling was a financially superior alternative to renting from the perspective of a marginal purchaser. The remaining 25.0% of months were in the capital-cost regime, during which expected appreciation—while still positive—had not fully offset the carrying cost. The 70.8% share of months with negative FUC (Table G.12) is slightly less than the 75.0% investment-regime share, because the investment regime requires only $\mathrm{FUC}_B \leq R^N$ (which can be satisfied even when FUC is moderately positive, as long as it remains below the prevailing rent rate), while a negative FUC requires that the user cost rate itself turns negative.

During the **Deflation episode** (1993–2003, $N = 132$ months), the regime reversed dramatically: 93.9% of months were classified as capital-cost regime. This is the strongest regime signal in the sample and underscores the asymmetry of housing market dynamics: bubbles are associated with investment-regime dominance, but they are followed by an even more protracted capital-cost regime as deflation compounds the carrying cost burden. The 0.0% negative-FUC share confirms that the OC approach in this episode simply coincided with the FUC: with no negative user costs to replace, the OC equalled the FUC throughout the Deflation.

The **Low-rate** (2014–20) and **Recent** (2021–25) episodes both show 100% investment-regime classification, but with different character. In the Low-rate episode, the near-zero prime rate and moderate appreciation collectively suppressed the FUC below the rent rate in every month, but with 28.6% of months recording a negative FUC, the margin was sometimes thin. In the Recent episode, the stronger appreciation (5.34% per annum) again drives investment-regime classification throughout, but the positive prime-rate trajectory (1.51% average, rising toward 2025) provides a natural floor that keeps the Recent episode's dynamics more moderate than the Bubble.

The full-sample row (55.0% investment, 45.0% capital-cost) reflects the averaging across these four sharply contrasting regimes and the transitional periods between them. It should be interpreted as a structural description of the Tokyo market rather than a stationary probability: the two regimes are not randomly distributed across the 480 months but are serially clustered into multi-year episodes whose transitions are linked to interest-rate cycles and asset-price dynamics.

Table G.14: Regime Classification: Opportunity Cost Approach, FUC Type B ($\lambda = 0.50$)

Episode	N	Investment	Capital-cost	$\text{FUC}_B < 0$ (%)
Bubble (1987–90)	48	75.0%	25.0%	70.8%
Deflation (1993–03)	132	6.1%	93.9%	0.0%
Low-rate (2014–20)	84	100.0%	0.0%	28.6%
Recent (2021–25)	60	100.0%	0.0%	0.0%
Full sample	480	55.0%	45.0%	—

Note: Investment regime: $\text{FUC}_B \leq R^N$. Capital-cost regime: $\text{FUC}_B > R^N$. $\text{FUC}_B < 0$: share of months with negative user cost rate $u_t^{f,B}$. Full-sample $N = 480$ includes transitional periods not assigned to a named episode.

G.6 Complete User Cost Rate Panel

Table G.15 provides the complete annual user cost rate panel for all nine approaches across the four market episodes and the full sample. Figures G.12 and G.13 present the time-series detail for the acquisitions and payments approaches, respectively, with sub-component decompositions.

Connection to the main text. Table 4 of the main text presents an identical table. Table G.15 here is retained in the supplementary appendix to serve as the single, complete numerical reference that readers can use when tracing the main-text narrative across multiple passages—for example, when the main text quotes "14.57 per cent" (FUC B, Deflation) or "21.56 per cent" (acquisitions, Bubble) without requiring the reader to locate the specific row of Table 4.

Detailed analysis: acquisitions approach (Figure G.12). Figure G.12 plots the acquisitions index P^H (Panel A) against the official CPI OOH, and the annualised log-change $\Delta \ln P^H$ (Panel B) over the full sample. Three features stand out. *Extreme amplitude:* The acquisitions rate swings from +21.56% per annum (Bubble) to −7.35% (Deflation)—a range of 28.91 percentage points within the span of two decades. No other approach exhibits volatility of this magnitude. *Procyclicality:* The acquisitions rate is highest exactly when the welfare-consistent FUC rate is most negative (Bubble) and lowest (negative) exactly when the FUC rate is highest (Deflation). This confirms the welfare–investment layer divergence (equation 3.19) and underscores the main text's argument that the acquisitions approach is "the approach most prone to procyclical mismeasurement" (Section 1). *CPI gap in the Recent episode:* In the post-2021 period, the acquisitions index averaged 117.6 against the official CPI OOH of 100.8—a gap of 16.8 index points—indicating that asset-price appreciation has once again outpaced the official shelter-cost measure, replicating the directional signal (though not the magnitude) of the Bubble episode.

Detailed analysis: payments approach (Figure G.13). Figure G.13 plots the payment rates

(Panel A), the payment monetary indices (Panel B), and the sub-component decomposition for Type B (Panel C). The payments approach is the only OOH measure that is always positive and anchored to observable cash flows (mortgage interest plus property tax), making it operationally attractive for statistical agencies. However, its fundamental limitation—ignoring expected capital gains—is visible in Panel A: during the Bubble, when the welfare-consistent cost of shelter was at its historical minimum, the payments rate for Type B stood at 4.73% (Table 4), far above the FUC of −4.06%. Panel C illustrates the decomposition for Type B: mortgage interest ($\lambda_B \times r_t^m \times P^H$) accounts for most of the variation, while the property tax component ($\tau \times P^H$) rises steadily with the asset price. The absence of an expected appreciation term means that the payments index cannot capture the investment-regime dynamics that are central to the OOH measurement problem.

Table G.15: Annual User Cost Rates (% p.a.) by Measurement Approach and Market Episode

Approach	Bubble	Deflation	Low-rate	Recent	Full
Rent-to-price (R/P)	1.92	3.45	2.63	2.26	2.97
Acquisitions ($\Delta \ln P^H$)	21.56	−7.35	4.81	7.28	2.76
Basic UC (HP-smoothed)	−5.31	13.32	−0.18	−0.43	4.02
Payments A ($\lambda = 0.20$)	2.73	2.06	1.71	1.80	2.04
Payments B ($\lambda = 0.50$)	4.73	3.04	2.17	2.40	3.00
Payments C ($\lambda = 0.80$)	6.73	4.02	2.63	3.01	3.96
FUC Type B ($\lambda = 0.50$)	−4.06	14.57	1.07	0.82	5.27
FUC Type C ($\lambda = 0.80$)	−4.51	14.12	0.62	0.37	4.82
FUC Type A ($\lambda = 0.20$)	−3.61	15.02	1.52	1.27	5.72

Note: Acquisitions rate = annualised $\Delta \ln P^H$. Basic UC and FUC use HP-smoothed expected appreciation ($\bar{\lambda}_{\text{HP}} = 129{,}600$). Payments rate = $\lambda_j r_t^m + \tau$ where $r_t^m = \text{prime} + 0.005$. All rates are annual averages for the indicated episode. Episode dates: Bubble 1987:01–1990:12; Deflation 1993:01–2003:12; Low-rate 2014:01–2020:12; Recent 2021:01–2025:12.

Figure G.14 plots the opportunity cost series $\text{OC}_t^j = \max\{\text{FUC}_t^j \cdot P_t^H,\ R_t^N\}$ for all three leverage types over the full sample, together with the new-contract rent R^N and the official CPI OOH. The figure provides a visual complement to Table G.14 and Figure 4 in the main text.

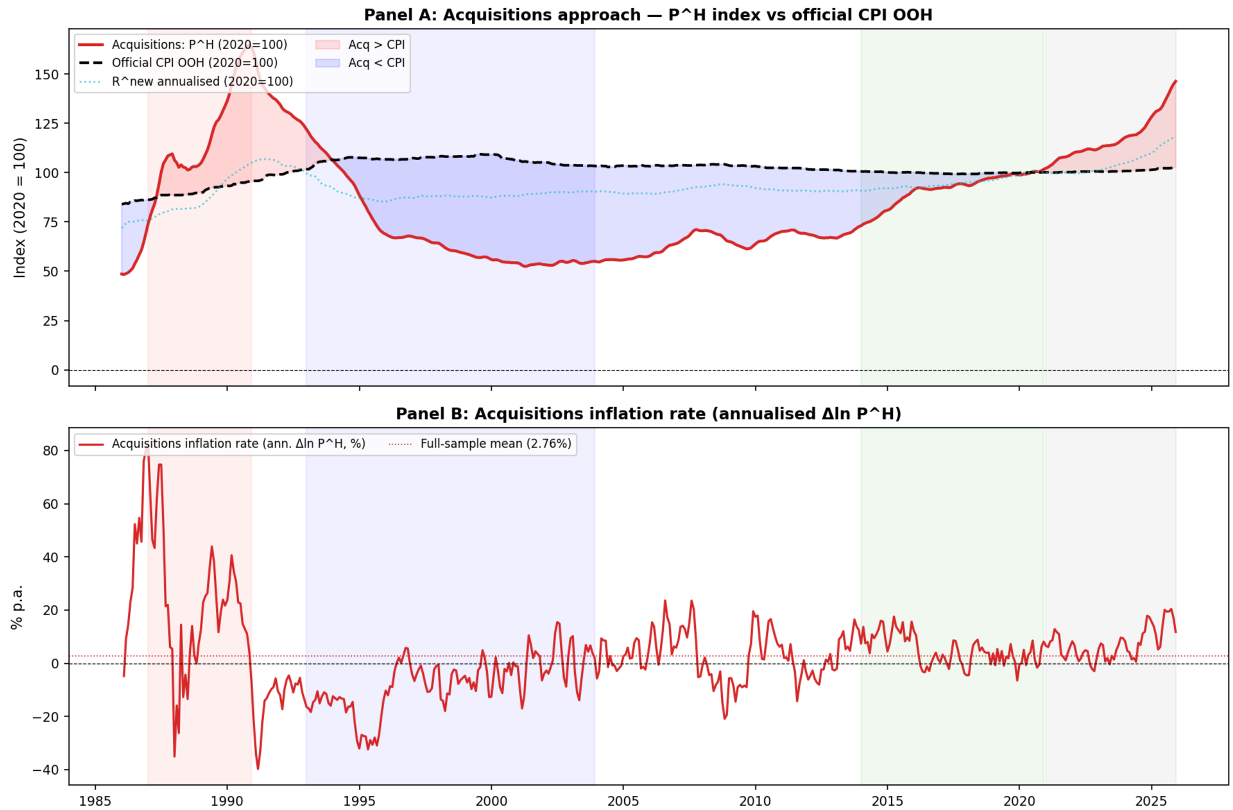

Figure G.12: Acquisitions approach, Tokyo 1986–2025. Panel A: Quality-adjusted price index P^H vs official CPI OOH (2020 = 100). The acquisitions index closely tracks the asset-price cycle, rising to 185 at the 1991 Bubble peak and falling below 70 at the 2004 Deflation trough, while the official CPI OOH—anchored by sticky incumbent rents—varied only between 93 and 109. Panel B: Annualised acquisitions inflation rate $\Delta \ln P^H$ (% p.a.). The rate swings from +22% (Bubble peak) to −16% (Deflation trough), confirming the procyclical character of the acquisitions approach relative to the welfare-consistent cost of shelter.

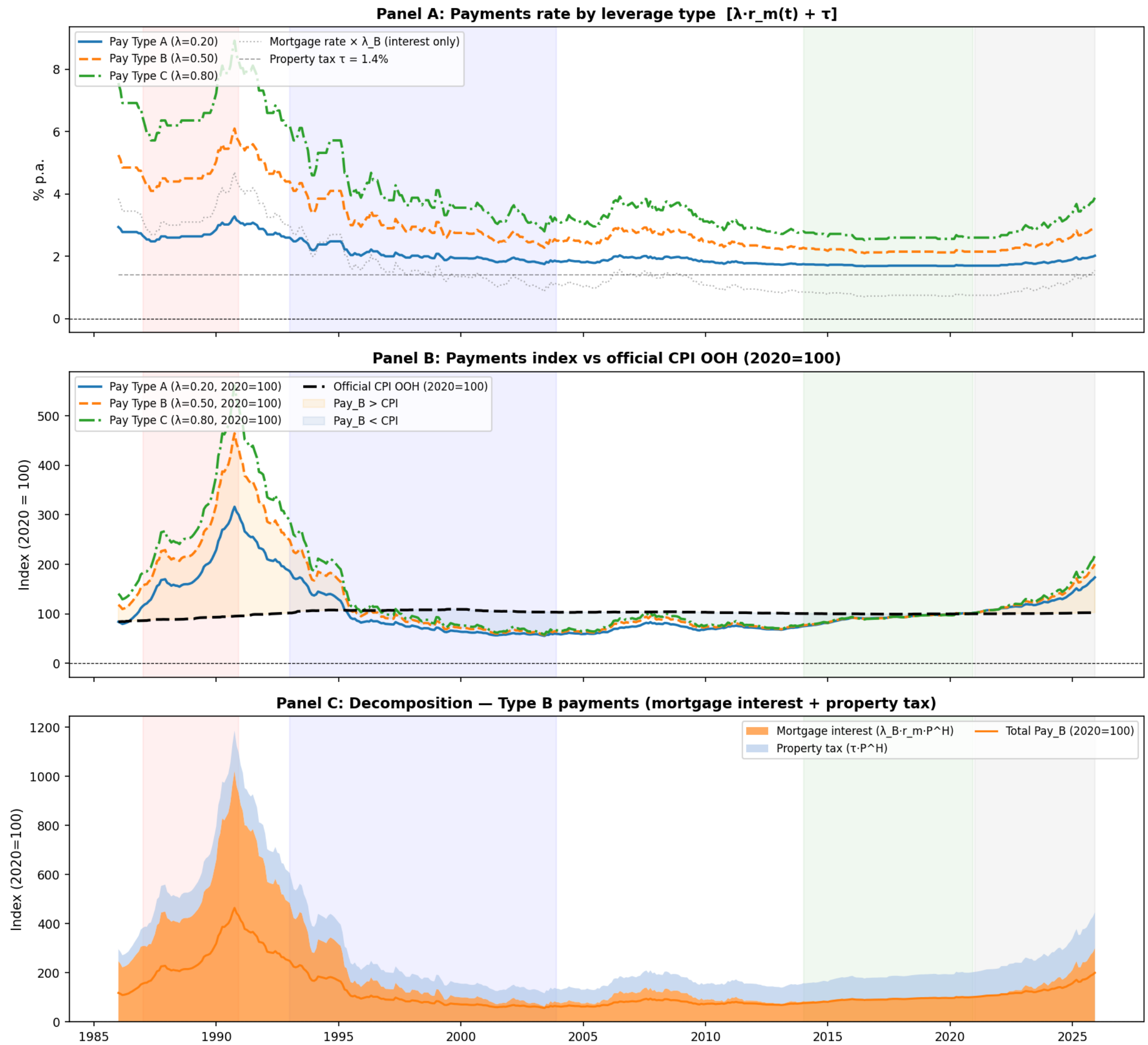

Figure G.13: Payments approach by leverage type, Tokyo 1986–2025. Panel A: Payment rates $\lambda r_t^m + \tau$ (% p.a.) for Types A, B, and C. The 0.90 pp spread between Types A and C is constant (as in the FUC), but the level is lower because the payments rate excludes the equity opportunity cost ρ^e and expected appreciation π_t^e. Panel B: Payment monetary indices ($2020 = 100$) vs official CPI OOH. All payment indices rise during the Bubble (when asset prices and mortgage rates were both high) and decline during the Deflation, producing a pattern broadly similar to the acquisitions index but less volatile. Panel C: Sub-component decomposition for Type B. The mortgage interest component ($\lambda_B r_t^m P^H$) accounts for the majority of variation, with the property tax component (τP^H) rising gradually as asset prices appreciate.

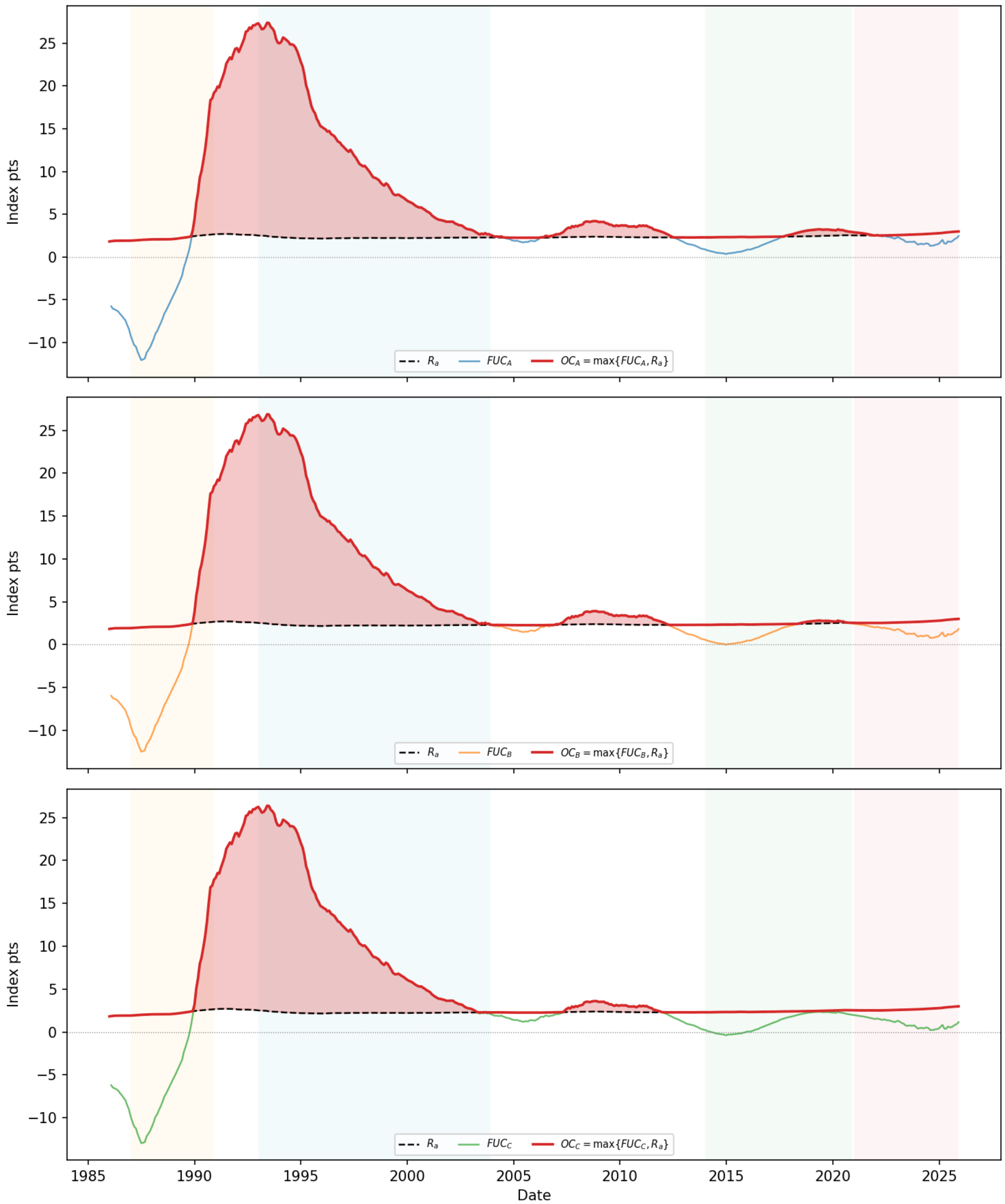

Figure G.14: Opportunity cost series $\text{OC}_t^j = \max\{\text{FUC}_t^j \cdot P_t^H,\ R_t^N\}$ for Types A, B, and C (2020 = 100), together with the new-contract rent R^N and official CPI OOH, Tokyo 1986–2025. The three OC series lie above the new-contract rent whenever the FUC monetary value exceeds the rent (capital-cost regime), and coincide with the rent whenever the FUC is below it (investment regime). In both regimes the OC is strictly positive, confirming the theoretical result of Section 3.4 and the empirical finding of Table 5 in the main text (zero negative months across all 480 observations). The spread among the three OC types narrows during investment-regime episodes—because all three are floored at R^N—and widens during capital-cost regimes, where the 0.90 pp financing-structure spread (Section G.3) is fully operative.

G.7 Depreciation Sensitivity and the Weight Ratio

Table G.16 examines the sensitivity of the user cost weight ratio V_U^0/V_A^0 to the structure depreciation rate δ, confirming the robustness of the near-unity result obtained with the full-sample Tokyo calibration.

Connection to the main text. Section 5.4 of the main text derives the weight ratio (equation 5.2) and reports that, calibrated with full-sample Tokyo estimates ($g = 2.87\%$, $r^* = 2.69\%$, $\delta = 0.020$), it yields $V_U^0/V_A^0 \approx 0.99$—close to unity and well below the theoretical baseline value of 1.68 reported in Table 1. The main text notes that "sensitivity analysis confirms the linearity of the user cost rate with respect to the depreciation rate: $\partial u_t/\partial\delta = 1$ in every period," and refers to Table G.16 for the corresponding changes in V_U^0/V_A^0.

Detailed analysis. The weight ratio V_U^0/V_A^0 varies only between 0.990 and 0.993 across the range $\delta \in [0.018, 0.022]$—a change of just 0.003 for a 10% variation in the depreciation rate. This near-zero sensitivity occurs because the weight ratio $(1 + g)(r^* + \delta)/(g + \delta)$ is driven primarily by the relationship between g and r^*, not by the level of δ. When $g \approx r^*$, as in the Tokyo full-sample calibration (2.87% vs 2.69%), the numerator $(r^* + \delta)$ and the denominator $(g + \delta)$ are nearly equal for any value of δ, making the ratio insensitive to depreciation uncertainty.

This finding supports the main text's conclusion that the dominant source of CPI measurement bias in Tokyo is the *price channel* (the procyclical acquisitions index and the lagging sticky-rent CPI) rather than the *weight channel* (the sub-unity weight ratio that the acquisitions approach assigns to housing services). Even if the depreciation rate were mis-specified by ±10%, the implied weight ratio would change by less than 0.5%, an order of magnitude smaller than the divergences in measured price documented in Sections 4.3–4.5.

The contrast with the theoretical baseline (Table 1: $V_U^0/V_A^0 = 1.68$ at $\delta = 0.020$, $r^* = 0.030$, $g = 0.010$) is instructive. In the baseline calibration, the real interest rate substantially exceeds the long-run appreciation rate ($r^* - g = 2.0$ pp), creating a large gap between user-cost value and acquisitions value. In the Tokyo data, this gap has been compressed by four decades of sustained asset-price appreciation tracking the prime rate closely. The implication for statistical agencies in different national contexts is that the weight-channel distortion from the acquisitions approach is largest precisely in economies where real interest rates are high and asset prices grow slowly—the opposite of the Tokyo configuration—and that country-specific calibration of V_U^0/V_A^0 is essential before concluding that the acquisitions approach's weight distortion is economically negligible.

Table G.16: Sensitivity of V_U^0/V_A^0 to the Depreciation Rate δ

	$\delta = 0.018$	$\delta = 0.020$	$\delta = 0.022$
V_U^0/V_A^0	0.990	0.991	0.993
Change from baseline	−0.001	—	+0.002

Note: Calibrated using full-sample Tokyo estimates: $g = 2.87\%$ p.a., $r^* = 2.69\%$ p.a. (mean prime rate). Baseline: $\delta = 0.020$. The near-zero sensitivity reflects the structural condition $g \approx r^*$ in Tokyo (2.87% vs 2.69%): when g and r^* are close, $V_U^0/V_A^0 \approx 1$ for any δ. Compare with the theoretical baseline (Table 1, main text): $g = 1\%$, $r^* = 3\%$ gives $V_U^0/V_A^0 = 1.68$.

Online Appendix References

References

[1] White, K.J. (2004), *Shazam: User's Reference Manual*, Version 10, Vancouver, Canada: Northwest Econometrics Ltd.

[2] Shimizu, C., K.G. Nishimura and T. Watanabe (2016), "House Prices at Different Stages of the Buying/Selling Process", *Regional Science and Urban Economics* 59, 37–53. http://dx.doi.org/10.1016/j.regsciurbeco.2016.04.001